Exercices sur la dérivation corrigé

Exercice 7

a) $f(x) = x^3 - 2x^2 - 4x$
dom f = dom f' = $\mathbb{R}$
$\left(\forall x \in \text{dom } f'\right) : f'(x) = 3x^2 - 2 \cdot 2x - 4 = 3x^2 - 4x - 4$

b) $f(x) = (5x+3)^3$
dom f = dom f' = $\mathbb{R}$
$\left(\forall x \in \text{dom } f'\right) : f'(x) = 3 \cdot (5x+3)^2 \cdot 5 = 15(5x+3)^2$

c) $f(x) = 2x^2 + x - 7$
dom f = dom f' = $\mathbb{R}$
$\left(\forall x \in \text{dom } f'\right) : f'(x) = 4x + 1$

d) $f(x) = \dfrac{2x+1}{1-3x}$
condition: $1 - 3x \neq 0 \Leftrightarrow 1 \neq 3x \Leftrightarrow x \neq \dfrac{1}{3}$
dom f = dom f' = $\mathbb{R} - \left\{\dfrac{1}{3}\right\}$
$\left(\forall x \in \text{dom } f'\right) : f'(x) = \dfrac{2(1-3x) - (2x+1) \cdot (-3)}{(1-3x)^2} = \dfrac{2 - 6x - (-6x - 3)}{(1-3x)^2} = \dfrac{2 - 6x + 6x + 3}{(1-3x)^2} = \dfrac{5}{(1-3x)^2}$

e) $f(x) = 3x^3 + \dfrac{1}{x}$
condition: $x \neq 0$
dom f = dom f' = $\mathbb{R}^*$
$\left(\forall x \in \text{dom } f'\right) : f'(x) = \dfrac{9x^4 - 1}{x^2}$

f) $f(x) = \sqrt{x^2 + 4x + 3}$
condition: $x^2 + 4x + 3 \geq 0$
$\Delta = 16 - 12 = 4; x_1 = \dfrac{-4+2}{2} = -1; x_2 = \dfrac{-4-2}{2} = -3$

x	$-\infty$		-3		-1		$+\infty$
x^2+4x+3		$+$	0	$-$	0	$+$	

dom f = $]-\infty;-3] \cup [-1;+\infty[$ et dom f' = $]-\infty;-3[\cup]-1;+\infty[$
$\left(\forall x \in \text{dom } f'\right) : f'(x) = \dfrac{2x+4}{2\sqrt{x^2+4x+3}} = \dfrac{2(x+2)}{2\sqrt{x^2+4x+3}} = \dfrac{x+2}{\sqrt{x^2+4x+3}}$

g) $f(x) = (x + \sqrt{x})^4$
condition: $x \geq 0$
dom f = $[0;+\infty[$; dom f' = $]0;+\infty[$
$\left(\forall x \in \text{dom } f'\right) : f'(x) = 4(x+\sqrt{x})^3 \cdot \left(1 + \dfrac{1}{2\sqrt{x}}\right) = 4(x+\sqrt{x})^3 \cdot \dfrac{2\sqrt{x}+1}{2\sqrt{x}} = \dfrac{2(x+\sqrt{x})^3 \cdot (2\sqrt{x}+1)}{\sqrt{x}}$

h) $f(x) = \dfrac{3}{(2x+1)^4}$

condition: $(2x+1)^4 \neq 0 \Leftrightarrow 2x+1 \neq 0 \Leftrightarrow 2x \neq -1 \Leftrightarrow x \neq -\dfrac{1}{2}$

$\text{dom } f = \text{dom } f' = \mathbb{R} - \left\{-\dfrac{1}{2}\right\}$

$f(x) = 3(2x+1)^{-4}$

$\left(\forall x \in \text{dom } f'\right) : f'(x) = 3 \cdot (-4) \cdot (2x+1)^{-5} \cdot 2 = -24 \cdot (2x+1)^{-5} = \dfrac{-24}{(2x+1)^5}$

i) $f(x) = \dfrac{x^2+3x-1}{x^2-4}$

condition: $x^2 - 4 \neq 0 \Leftrightarrow (x-2)(x+2) \neq 0 \Leftrightarrow x \neq 2$ et $x \neq -2$

$\text{dom } f = \text{dom } f = \mathbb{R} - \{-2; 2\}$

$\left(\forall x \in \text{dom } f\right) : f'(x) = \dfrac{(2x+3)(x^2-4)-(x^2+3x-1)(2x)}{(x^2-4)^2} = \dfrac{(2x^3+3x^2-8x-12)-(2x^3+6x^2-2x)}{(x^2-4)^2}$

$= \dfrac{2x^3+3x^2-8x-12-2x^3-6x^2+2x}{(x^2-4)^2} = \dfrac{-3x^2-6x-12}{(x^2-4)^2}$

j) $f(x) = x^2 \cdot \sqrt{x+1}$

condition: $x+1 \geq 0 \Leftrightarrow x \geq -1$

$\text{dom } f = [-1; +\infty[; \text{dom } f' =]-1; +\infty[$

$\left(\forall x \in \text{dom } f'\right) : f'(x) = 2x \cdot \sqrt{x+1} + x^2 \cdot \dfrac{1}{2\sqrt{x+1}} = 2x\sqrt{x+1} + \dfrac{x^2}{2\sqrt{x+1}} = \dfrac{2x(x+1)}{2\sqrt{x+1}} + \dfrac{x^2}{2\sqrt{x+1}}$

$= \dfrac{2x^2+2x+x^2}{2\sqrt{x+1}} = \dfrac{3x^2+2x}{2\sqrt{x+1}}$

k) $f(x) = \dfrac{(2x+1)^2}{1-4x}$

condition: $1-4x \neq 0 \Leftrightarrow 4x \neq 1 \Leftrightarrow x \neq \dfrac{1}{4}$

$\text{dom } f = \text{dom } f' = \mathbb{R} - \left\{\dfrac{1}{4}\right\}$

$\left(\forall x \in \text{dom } f'\right) : f'(x) = \dfrac{2(2x+1) \cdot 2 \cdot (1-4x) - (2x+1)^2(-4)}{(1-4x)^2} = \dfrac{4(2x+1)[(1-4x)+(2x+1)]}{(1-4x)^2}$

$\dfrac{(8x+4)(-2x+2)}{(1-4x)^2} = \dfrac{-16x^2+16x-8x+8}{(1-4x)^2} = \dfrac{-16x^2+8x+8}{(1-4x)^2}$

l) $f(x) = (4x+1)^2(2-3x)^3$

$\text{dom } f = \text{dom } f' = \mathbb{R}$

$\left(\forall x \in \text{dom } f'\right) : f'(x) = 2(4x+1) \cdot 4 \cdot (2-3x)^3 + (4x+1)^2 \cdot 3(2-3x)^2 \cdot (-3) = (4x+1)(2-3x)^2[8(2-3x) - 9(4x+$

$= (4x+1)(2-3x)^2(16-24x-36x-9) = (4x+1)(2-3x)^2(-60x+7)$

m) $f(x) = \dfrac{4x}{\sqrt{x^2-9}}$

condition: $x^2 - 9 \geq 0$

$x^2 - 9 = 0 \Leftrightarrow (x-3)(x+3) = 0 \Leftrightarrow x = 3$ ou $x = -3$

x	$-\infty$		-3		3		$+\infty$
x^2+4x+3		$+$	0	$-$	0	$+$	

dom f $=]-\infty;-3] \cup [3;+\infty[$ et dom f' $=]-\infty;-3[\cup]3;+\infty[$

$$\left(\forall x \in \text{dom } f'\right) : f'(x) = \frac{4\sqrt{x^2-9} - 4x \cdot \frac{2x}{2\sqrt{x^2-9}}}{\left(\sqrt{x^2-9}\right)^2} = \frac{4\sqrt{x^2-9} - \frac{4x^2}{\sqrt{x^2-9}}}{x^2-9} = \frac{\frac{4\left(x^2-9\right)-4x^2}{\sqrt{x^2-9}}}{x^2-9} = \frac{4x^2-36-4x^2}{(x^2-9)\sqrt{x^2-9}} = \frac{-36}{(x^2-9)\sqrt{x^2-9}}$$

n) $f(x) = \dfrac{4x+1}{2x^2+5x-3}$

condition: $2x^2 + 5x - 3 \neq 0$,

$\Delta = 25 + 24 = 49; x_1 = \dfrac{-5+7}{4} = \dfrac{1}{2}; x_2 = \dfrac{-5-7}{4} = -3$

dom f $=$ dom f' $= \mathbb{R} - \left\{-3; \dfrac{1}{2}\right\}$

$$\left(\forall x \in \text{dom } f'\right) : f'(x) = \frac{4(2x^2+5x-3)-(4x+1)(4x+5)}{(2x^2+5x-3)^2} = \frac{8x^2+20x-12-(16x^2+20x+4x+5)}{(2x^2+5x-3)^2}$$

$$= \frac{8x^2+20x-12-16x^2-24x-5}{(2x^2+5x-3)^2} = \frac{-8x^2-4x-17}{(2x^2+5x-3)^2}$$

o) $f(x) = x + \dfrac{2x}{x} - \dfrac{2}{9x^2}$

condition: $x \neq 0$

dom f $=$ dom f' $= \mathbb{R}^*$

$f(x) = x + 2 - \dfrac{2}{9x}$

$$\left(\forall x \in \text{dom } f'\right) : f'(x) = 1 - \frac{2}{9} \cdot \left(-\frac{1}{x^2}\right) = 1 + \frac{2}{9x^2} = \frac{9x^2+2}{9x^2}$$

Exercice 8

a) $f(x) = -2x + 1$

• $\text{dom } f = \text{dom } f' = \mathbb{R}$

• $\lim\limits_{x\to\pm\infty} f(x) = \lim\limits_{x\to\pm\infty} (-2x) = \mp\infty$

• $(\forall x \in \text{dom } f') : f'(x) = -2 < 0$

x	$-\infty$ $\qquad$ $+\infty$
f'	$-$
f	$+\infty$ $\searrow$ $-\infty$

b) $f(x) = -x^2 + 6x - 5$

• $\text{dom } f = \text{dom } f' = \mathbb{R}$

• $\lim\limits_{x\to\pm\infty} f(x) = \lim\limits_{x\to\pm\infty} (-x^2) = -\infty$

• $(\forall x \in \text{dom } f') : f'(x) = -2x + 6$

$f'(x) = 0 \Leftrightarrow -2x + 6 = 0 \Leftrightarrow x = 3$

tableau des variations

x	$-\infty$		3		$+\infty$
f'		$+$	0	$-$	
f	$-\infty$	$\nearrow$	4	$\searrow$	$-\infty$

$f(3) = -9 + 18 - 5 = 4$

c) $f(x) = x^3 + 3x^2 - 9x + 1$

•$\text{dom } f = \text{dom } f' = \mathbb{R}$

• $\lim\limits_{x\to\pm\infty} f(x) = \lim\limits_{x\to\pm\infty} (x^3) = \pm\infty$

• $\left(\forall x \in \text{dom } f'\right) : f'(x) = 3x^2 + 6x - 9$

$f'(x) = 0 \Leftrightarrow 3x^2 + 6x - 9 = 0 \Leftrightarrow x^2 + 2x - 3 = 0 \overset{\Delta=16}{\Leftrightarrow} x = -3 \text{ ou } x = 1$

signe de la dérivée:

x	$-\infty$		-3		1		$+\infty$
$f'(x)$		$+$	0	$-$	0	$+$	

tableau des variations

x	$-\infty$		-3		1		$+\infty$
f'		$+$	0	$-$	0	$+$	
f	$-\infty$	$\nearrow$	28	$\searrow$	5	$\nearrow$	$+\infty$

$f(-3) = -27 + 27 + 27 + 1 = 28; f(1) = 1 + 3 - 9 + 1 = 5$

d) $f(x) = 2x^3 - 3x^2 - 36x + 15$

•$\text{dom } f = \text{dom } f' = \mathbb{R}$

• $\lim\limits_{x\to\pm\infty} f(x) = \lim\limits_{x\to\pm\infty} (2x^3) = \pm\infty$

• $\left(\forall x \in \text{dom } f'\right) : f'(x) = 6x^2 - 6x - 36$

$f'(x) = 0 \Leftrightarrow 6x^2 - 6x - 36 = 0 \Leftrightarrow x^2 - x - 6 = 0 \overset{\Delta=25}{\Leftrightarrow} x = 3 \text{ ou } x = -2$

signe de la dérivée:

x	$-\infty$		-2		3		$+\infty$
$f'(x)$		$+$	0	$-$	0	$+$	

tableau des variations

x	$-\infty$		-2		3		$+\infty$
f'		$+$	0	$-$	0	$+$	
f	$-\infty$	$\nearrow$	59	$\searrow$	-66	$\nearrow$	$+\infty$

$f(3) = 54 - 27 - 108 + 15 = -66; f(-2) = -16 - 12 + 72 + 15 = 59$

e) $f(x) = x^4 - 32x^2$

•dom f = dom $f' = \mathbb{R}$

•$\lim\limits_{x\to\pm\infty} f(x) = \lim\limits_{x\to\pm\infty} (x^4) = +\infty$

•$\left(\forall x \in \text{dom } f'\right)$: $f'(x) = 4x^3 - 64x$

$f'(x) = 0 \Leftrightarrow 4x^3 - 64x = 0 \Leftrightarrow 4x(x^2 - 16) = 0 \Leftrightarrow 4x(x-4)(x+4) = 0 \Leftrightarrow x = 0$ ou $x = 4$ ou $x = -4$

signe de la dérivée:

x	$-\infty$		-4		0		4		$+\infty$
$4x$		$-$		$-$	0	$+$		$+$	
$x^2 - 16$		$+$	0	$-$		$-$	0	$+$	
$f'(x)$		$-$	0	$+$	0	$-$	0	$+$	

tableau des variations

x	$-\infty$		-4		0		4		$+\infty$
f'		$-$	0	$+$	0	$-$	0	$+$	
f	$+\infty$	$\searrow$	-256	$\nearrow$	0	$\searrow$	-256	$\nearrow$	$+\infty$

$f(-4) = 256 - 512 = -256; f(0) = 0; f(4) = 256 - 512 = -256$

f) $f(x) = 5x^6 - 6x^5 - 15x^4$

•dom f = dom $f' = \mathbb{R}$

•$\lim\limits_{x\to\pm\infty} f(x) = \lim\limits_{x\to\pm\infty} (5x^6) = +\infty$

•$\left(\forall x \in \text{dom } f'\right)$: $f'(x) = 30x^5 - 30x^4 - 60x^3$

•$f'(x) = 30x^5 - 30x^4 - 60x^3 = 0 \Leftrightarrow 30x^3(x^2 - x - 2) = 0 \Leftrightarrow 30x^3 = 0$ ou $x^2 - x - 2 = 0 \overset{\Delta=9}{\Leftrightarrow} x = 0$ ou $x = 2$ ou $x = -1$

signe de la dérivée:

x	$-\infty$		-1		0		2		$+\infty$
$30x^3$		$-$		$-$	0	$+$		$+$	
$x^2 - x - 2$		$+$	0	$-$		$-$	0	$+$	
$f'(x)$		$-$	0	$+$	0	$-$	0	$+$	

tableau des variations

x	$-\infty$		-1		0		2		$+\infty$
f'		$-$	0	$+$	0	$-$	0	$+$	
f	$+\infty$	$\searrow$	-112	$\nearrow$	0	$\searrow$	-4	$\nearrow$	$+\infty$

$f(2) = 5 \cdot 64 - 6 \cdot 32 - 15 \cdot 16 = -112 = -112; f(0) = 0; f(-1) = 5 + 6 - 15 = -4$

g) $f(x) = \dfrac{x^2}{x+1}$

•cond.:$x + 1 \neq 0 \Leftrightarrow x \neq -1$

•dom f = dom f$' = \mathbb{R} \setminus \{-1\}$

• $\lim\limits_{x\to\pm\infty} f(x) = \lim\limits_{x\to\pm\infty} \dfrac{x^2}{x} = \lim\limits_{x\to\pm\infty} x = \pm\infty$

• $\lim\limits_{x\to-1} \dfrac{\overbrace{x^2}^{\to 1}}{\underbrace{x+1}_{\to 0}}$ il faut distinguer à gauche et à droite

x	$-\infty$		-1		$+\infty$
$x+1$		$-$	0	$+$	

$\lim\limits_{x\to-1^+} \dfrac{\overbrace{x^2}^{\to 1}}{\underbrace{x+1}_{\to 0^+}} = +\infty \qquad \lim\limits_{x\to-1^-} \dfrac{\overbrace{x^2}^{\to 1}}{\underbrace{x+1}_{\to 0^-}} = -\infty \qquad$ A.V. : $x = -1$

•$(\forall x \in \text{dom } f') : f'(x) = \dfrac{2x(x+1) - x^2 \cdot 1}{(x+1)^2} = \dfrac{2x^2 + 2x - x^2}{(x+1)^2} = \dfrac{x^2 + 2x}{(x+1)^2}$

•$f'(x) = 0 \Leftrightarrow x^2 + 2x = 0 \Leftrightarrow x(x+2) = 0 \Leftrightarrow x = 0$ ou $x = -2$

x	$-\infty$		-2		0		$+\infty$
x^2+2x		$+$	0	$-$	0	$+$	

tableau des variations

x	$-\infty$		-2		-1		0		$+\infty$
f'		$+$	0	$-$	$\Vert$	$-$	0	$+$	
f	$-\infty$	$\nearrow$	-4	$\searrow$	$-\infty \Vert +\infty$	$\searrow$	0	$\nearrow$	$+\infty$

$f(-2) = \frac{4}{-1} = -4; f(0) = 0$

h) $f(x) = \dfrac{3x+1}{4x-3}$

•cond.: $4x - 3 \neq 0 \Leftrightarrow 4x \neq 3 \Leftrightarrow x \neq \dfrac{3}{4}$

•dom f = dom f$' = \mathbb{R} \setminus \left\{\frac{3}{4}\right\}$

• $\lim\limits_{x\to\pm\infty} f(x) = \lim\limits_{x\to\pm\infty} \dfrac{3x}{4x} = \dfrac{3}{4} \qquad$ A.H. : $y = \dfrac{3}{4}$

• $\lim\limits_{x\to\frac{3}{4}} \dfrac{\overbrace{3x+1}^{\to \frac{13}{4}}}{\underbrace{4x-3}_{\to 0}}$ il faut distinguer à gauche et à droite

x	$-\infty$		$\frac{3}{4}$		$+\infty$
$4x-3$		$-$	0	$+$	

$\lim\limits_{x\to\frac{3}{4}^+} \dfrac{\overbrace{3x+1}^{\to \frac{13}{4}}}{\underbrace{4x-3}_{\to 0^+}} = +\infty \qquad \lim\limits_{x\to\frac{3}{4}^-} \dfrac{\overbrace{3x+1}^{\to \frac{13}{4}}}{\underbrace{4x-3}_{\to 0^-}} = -\infty \qquad$ A.V. : $x = \dfrac{3}{4}$

•$(\forall x \in \text{dom } f') : f'(x) = \dfrac{3(4x-3) - (3x+1) \cdot 4}{(4x-3)^2} = \dfrac{12x - 9 - 12x - 4}{(4x-3)^2} = \dfrac{-13}{(4x-3)^2} < 0$

tableau des variations

x	$-\infty$		$\frac{3}{4}$		$+\infty$
f'		$-$	0	$-$	
f	$\frac{3}{4}$	$\searrow$	$-\infty \Vert +\infty$	$\searrow$	$\frac{3}{4}$

i) $f(x) = \dfrac{x^2-1}{x^2+1}$

•cond: $x^2+1 \neq 0$ toujours vrai!

•dom f = dom f' = $\mathbb{R}$

• $\lim\limits_{x\to\pm\infty} f(x) = \lim\limits_{x\to\pm\infty} \dfrac{x^2}{x^2} = 1$ A.H.: $y = 1$

•$(\forall x \in \text{dom } f')$: $f'(x) = \dfrac{2x(x^2+1)-(x^2-1)\cdot 2x}{(x^2+1)^2} = \dfrac{2x^3+2x-2x^3+2x}{(x^2+1)^2} = \dfrac{4x}{(x^2+1)^2}$

x	$-\infty$		0		$+\infty$
4x		$-$	0	$+$	

tableau des variations

x	$-\infty$		0		$+\infty$
f'		$-$	0	$+$	
f	1	$\searrow$	-1	$\nearrow$	1

j) $f(x) = \dfrac{x}{1+x^2}$

•cond: $x^2+1 \neq 0$ toujours vrai!

•dom f = dom f' = $\mathbb{R}$

• $\lim\limits_{x\to\pm\infty} f(x) = \lim\limits_{x\to\pm\infty} \dfrac{x}{x^2} = \lim\limits_{x\to-\infty} \dfrac{1}{x} = 0$ A.H.: $y = 0$

•$(\forall x \in \text{dom } f')$: $f'(x) = \dfrac{1\cdot(1+x^2)-x\cdot 2x}{(1+x^2)^2} = \dfrac{1+x^2-2x^2}{(1+x^2)^2} = \dfrac{1-x^2}{(1+x^2)^2}$

•$f'(x) = 0 \Leftrightarrow 1-x^2 = 0 \Leftrightarrow (1-x)(1+x) = 0 \Leftrightarrow x = 1$ ou $x = -1$

x	$-\infty$		-1		1		$+\infty$
$1-x^2$		$-$	0	$+$	0	$-$	

tableau des variations

x	$-\infty$		-1		1		$+\infty$
f'		$-$	0	$+$	0	$-$	
f	0	$\searrow$	$-\frac{1}{2}$	$\nearrow$	$\frac{1}{2}$	$\searrow$	0

$f(-1) = \frac{-1}{2} = -\frac{1}{2}; f(1) = \frac{1}{2}$

k) $f(x) = \dfrac{2x^2-10x+18}{x-5}$

•dom f = dom f' = $\mathbb{R} \setminus \{5\}$

• $\lim\limits_{x\to\pm\infty} f(x) = \lim\limits_{x\to\pm\infty} \dfrac{2x^2}{x} = \lim\limits_{x\pm\infty} 2x = \pm\infty$

• $\lim\limits_{x\to 5} \dfrac{\overbrace{2x^2-10x+18}^{\to 18}}{\underbrace{x-5}_{\to 0}}$ il faut distinguer à gauche et à droite

x	$-\infty$		5		$+\infty$
$x-5$		$-$	0	$+$	

$\lim\limits_{x\to 5^+} \dfrac{\overbrace{2x^2-10x+18}^{\to 18}}{\underbrace{x-5}_{\to 0^+}} = +\infty$ et $\lim\limits_{x\to 5^-} \dfrac{\overbrace{2x^2-10x+18}^{\to 18}}{\underbrace{x-5}_{\to 0^-}} = -\infty$ A.V.: $x = 5$

•$(\forall x \in \text{dom } f')$: $f'(x) = \dfrac{(4x-10)(x-5)-(2x^2-10x+18)\cdot 1}{(x-5)^2}$

$= \dfrac{4x^2-20x-10x+50-2x^2+10x-18}{(x-5)^2} = \dfrac{2x^2-20x+32}{(x-5)^2}$

$\bullet f'(x)=0 \Leftrightarrow 2x^2-20x+32=0 \overset{\Delta=144}{\Leftrightarrow} x=8 \text{ ou } x=2$

x	$-\infty$		2		8		$+\infty$
$2x^2-20x+32$		+	0	−	0	+	

tableau des variations

x	$-\infty$		2		5		8		$+\infty$
f'		+	0	−	$\Vert$	−	0	+	
f	$-\infty$	$\nearrow$	-2	$\searrow$	$-\infty \Vert +\infty$	$\searrow$	22	$\nearrow$	$+\infty$

$f(2)=\frac{8-20+18}{2-5}=-2; f(8)=\frac{128-80+18}{8-5}=22$

l) $f(x)=\dfrac{-x^2+x+4}{x^2-x-2}$

$\bullet$cond.: $x^2-x-2\neq 0 \overset{\Delta=9}{\Leftrightarrow} x\neq \dfrac{1+3}{2}=2 \text{ et } x\neq \dfrac{1-3}{2}=-1$ donc $\text{dom } f=\text{dom } f'=\mathbb{R}\setminus\{-1;2\}$

$\bullet \lim\limits_{x\to\pm\infty} f(x)=\lim\limits_{x\to\pm\infty}\dfrac{-x^2}{x^2}=-1$ A.H.: $y=-1$

$\bullet \lim\limits_{x\to-1}\dfrac{\overbrace{-x^2+x+4}^{\to 2}}{\underbrace{x^2-x-2}_{\to 0}}$ il faut distinguer -1^+ et -1^-

x	$-\infty$		-1		2		$+\infty$
x^2-x-2		+	0	−	0	+	

$\lim\limits_{x\to-1^-}\dfrac{\overbrace{-x^2+x+4}^{\to 2}}{\underbrace{x^2-x-2}_{\to 0^+}}=+\infty$ et $\lim\limits_{x\to-1^+}\dfrac{\overbrace{-x^2+x+4}^{\to 2}}{\underbrace{x^2-x-2}_{\to 0^-}}=-\infty$ A.V.: $x=-1$

$\bullet \lim\limits_{x\to 2}\dfrac{\overbrace{-x^2+x+4}^{\to 2}}{\underbrace{x^2-x-2}_{\to 0}}$ il faut distinguer 2^+ et 2^-

$\lim\limits_{x\to 2^-}\dfrac{\overbrace{-x^2+x+4}^{\to 2}}{\underbrace{x^2-x-2}_{\to 0^-}}=-\infty$ et $\lim\limits_{x\to 2^+}\dfrac{\overbrace{-x^2+x+4}^{\to 2}}{\underbrace{x^2-x-2}_{\to 0^+}}=+\infty$ A.V.: $x=2$

$$\bullet(\forall x\in \text{dom}f'):\ f'(x)=\frac{(-2x+1)\cdot(x^2-x-2)-(-x^2+x+4)\cdot(2x-1)}{(x^2-x-2)^2}$$

$$=\frac{-2x^3+2x^2+4x+x^2-x-2-(-2x^3+x^2+2x^2-x+8x-4)}{(x^2-x-2)^2}=\frac{-2x^3+3x^2+3x-2+2x^3-3x^2-7x+4}{(x^2-x-2)^2}$$

$$=\frac{-4x+2}{(x^2-x-2)^2}$$

$\bullet f'(x)=0 \Leftrightarrow -4x+2=0 \Leftrightarrow x=\frac{1}{2}$

x	$-\infty$		$\frac{1}{2}$		$+\infty$
$-4x+2$		+	0	−	

tableau de variation

x	$-\infty$		-1		$\frac{1}{2}$		2		$+\infty$
f'		+	$\Vert$	+	0	−	$\Vert$	−	
f	-1	$\nearrow$	$+\infty \Vert -\infty$	$\nearrow$	$-\frac{17}{9}$	$\searrow$	$-\infty \Vert +\infty$	$\searrow$	-1

$f\left(\frac{1}{2}\right)=\dfrac{-\frac{1}{4}+\frac{1}{2}+4}{\frac{1}{4}-\frac{1}{2}-2}=-\frac{17}{9}\simeq -1{,}9$

Exercice 9

a) $f(x) = \dfrac{x^2 - x - 6}{x^2 + x - 6}$

$D_f = \mathbb{R} \setminus \{-3;2\}$

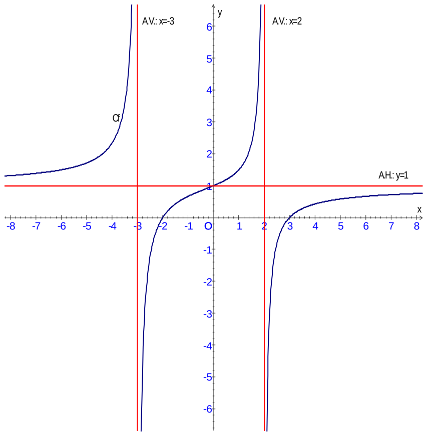
A.H.: $y = 1$; A.V.: $x = -3$; A.V.: $x = 2$

$f'(x) = \dfrac{2x^2 + 12}{(x^2 + x - 6)^2}$

$C_f \cap (Ox) = \{(-2;0);(3;0)\}$

$C_f \cap (Oy) = \{(0;1)\}$

x	$-\infty$		-3		2		$+\infty$
f'		$+$	$\|$	$+$	$\|$	$+$	
f	1	↗	$+\infty \;\|\; -\infty$	↗	$+\infty \;\|\; -\infty$	↗	1



b) $f(x) = \dfrac{x^2 - 1}{(x-2)^2}$

$D_f = \mathbb{R} \setminus \{2\}$

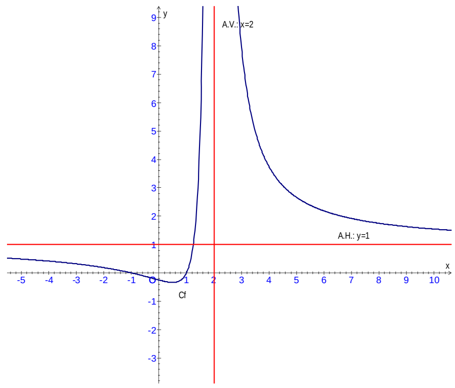
A.H.: $y = 1$; A.V.: $x = 2$

$f'(x) = \dfrac{-4x^2 + 10x - 4}{(x-2)^4}$

$C_f \cap (Ox) = \{(-1;0);(1;0)\}$

$C_f \cap (Oy) = \left\{\left(0;-\dfrac{1}{4}\right)\right\}$

x	$-\infty$		$\frac{1}{2}$		2		$+\infty$
f'		$-$	0	$+$	$\|$	$-$	
f	1	↘	$-\frac{1}{3}$	↗	$+\infty \;\|\; +\infty$	↘	1



c) $f(x) = \dfrac{x^2 - 2x + 1}{x^2 - 3x}$

$D_f = \mathbb{R} \setminus \{0;3\}$

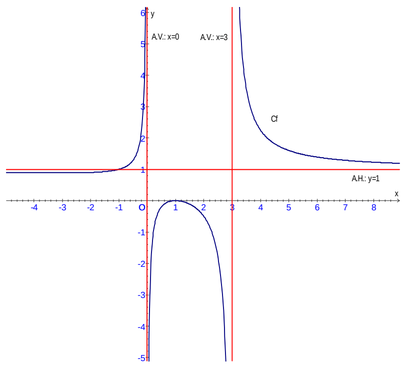
A.H.: $y = 1$; A.V.: $x = 0$; A.V.: $x = 3$

$f'(x) = \dfrac{-x^2 - 2x + 3}{(x^2 - 3x)^2}$

$C_f \cap (Ox) = \{(1;0)\}$

$C_f \cap (Oy) = \varnothing$

x	$-\infty$		-3		1		$+\infty$
$-x^2 - 2x + 3$		$-$	0	$+$	0	$-$	



d) $f(x) = \dfrac{-3x}{x^2 - 2x + 4}$

$D_f = \mathbb{R}$

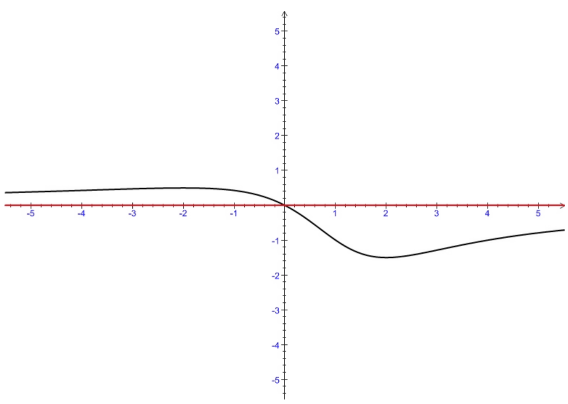
A.H.: $y = 0$

$f'(x) = \dfrac{3x^2 - 12}{(x^2 - 2x + 4)^2}$

$C_f \cap (Ox) = \{(0;0)\}$

$C_f \cap (Oy) = \{(0;0)\}$

x	$-\infty$		-2		2		$+\infty$
f'		$+$	0	$-$	0	$+$	
f	0	↗	$\frac{1}{2}$	↘	$-\frac{3}{2}$	↗	0



e) $f(x)=\dfrac{x^2-2x+6}{x+1}$

$D_f=\mathbb{R}\setminus\{-1\}$

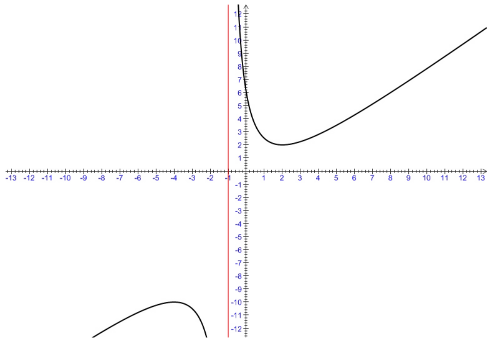
$f'(x)=\dfrac{x^2+2x-8}{(x+1)^2}$

A.V.: $x=-1$

$C_f\cap(Ox)=\varnothing$

$C_f\cap(Oy)=\{(0;6)\}$

x	$-\infty$		-4		-1		2		$+\infty$
f'		$+$	0	$-$	$\Vert$	$-$	0	$+$	
f	$-\infty$	$\nearrow$	-10	$\searrow$	$-\infty \Vert +\infty$	$\searrow$	2	$\nearrow$	$+\infty$



f) $f(x)=\dfrac{-x-3}{3x+2}$;

$D_f=\mathbb{R}\setminus\left\{-\frac{2}{3}\right\}$

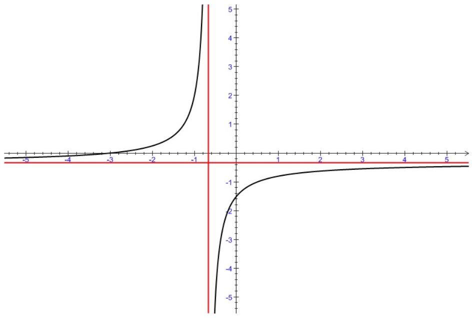
$f'(x)=\dfrac{7}{(3x+2)^2}$

A.H.: $y=-\frac{1}{3}$; A.V.: $x=-\frac{2}{3}$

$C_f\cap(Ox)=\{(-3;0)\}$

$C_f\cap(Oy)=\left\{\left(0;-\dfrac{3}{2}\right)\right\}$

x	$-\infty$		$-\frac{2}{3}$		$+\infty$
f'		$+$	$\Vert$	$+$	
f	$-\frac{1}{3}$	$\nearrow$	$+\infty \Vert -\infty$	$\nearrow$	$-\frac{1}{3}$



g) $f(x)=\dfrac{x+1}{x^2-4}$

$D_f=\mathbb{R}\setminus\{-2;2\}$

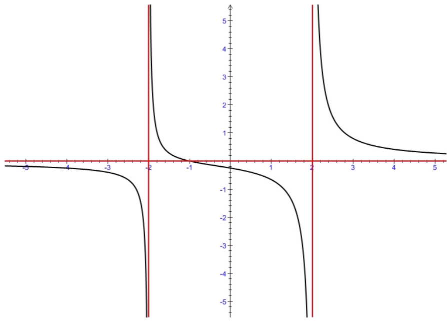
$f'(x)=-\dfrac{x^2+2x+4}{(x^2-4)^2}$

A.H.: $y=0$; A.V.: $x=-2$; A.V.: $x=-2$

$C_f\cap(Ox)=\{(-1;0)\}$

$C_f\cap(Oy)=\left\{\left(0;-\dfrac{1}{4}\right)\right\}$

x	$-\infty$		-2		2		$+\infty$
f'		$-$	$\Vert$	$-$	$\Vert$	$-$	
f	0	$\searrow$	$-\infty \Vert +\infty$	$\searrow$	$-\infty \Vert +\infty$	$\searrow$	0



h) $f(x)=\dfrac{3x^2+3x-1}{x^2+x}$

$D_f=\mathbb{R}\setminus\{-1;0\}$

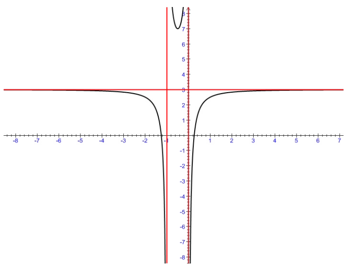
A.H.: $y=3$; A.V.: $x=-1$; A.V.: $x=0$

$f'(x)=\dfrac{2x+1}{(x^2+x)^2}$

$C_f\cap(Ox)=\left\{\left(\frac{-3-\sqrt{21}}{6};0\right);\left(\frac{-3+\sqrt{21}}{6};0\right)\right\}$

$C_f\cap(Oy)=\varnothing$

x	$-\infty$		-1		$-\frac{1}{2}$		0		$+\infty$
f'		$-$	$\Vert$	$-$		$+$	$\Vert$	$+$	
f	3	$\searrow$	$-\infty \Vert +\infty$	$\searrow$	7	$\nearrow$	$+\infty \Vert -\infty$	$\nearrow$	3



i) $f(x)=\dfrac{x^2-4x}{x^2+2x-3}$

$D_f=\mathbb{R}\setminus\{-3;1\}$

A.H.: $y=1$; A.V.: $x=-3$; A.V.: $x=1$

$f'(x)=\dfrac{6x^2-6x+12}{(x^2+2x-3)^2}$

$C_f\cap(Ox)=\{(0;0);(4;0)\}$

$C_f\cap(Oy)=\{(0;0)\}$

x	$-\infty$		-3		1		$+\infty$
f'		$+$	$\Vert$	$+$	$\Vert$	$+$	
f	1	$\nearrow$	$+\infty \Vert -\infty$	$\nearrow$	$+\infty \Vert -\infty$	$\nearrow$	1