Exercices sur la dérivation - étape 1 - corrigé

a) $f(x) = x^3 - 2x^2 - 4x + \cos(2\pi)$

$D_f = D_{f'} = \mathbb{R}$

$(\forall x \in D_{f'}) : f'(x) = 3x^2 - 2 \cdot 2x - 4 = 3x^2 - 4x - 4$

b) $f(x) = (5x+3)^3$

$D_f = D_{f'} = \mathbb{R}$

$(\forall x \in D_{f'}) : f'(x) = 3 \cdot (5x+3)^2 \cdot 5 = 15(5x+3)^2$

c) $f(x) = \dfrac{2x+1}{1-3x}$

condition: $1 - 3x \neq 0 \Leftrightarrow 1 \neq 3x \Leftrightarrow x \neq \dfrac{1}{3}$

$D_f = D_{f'} = \mathbb{R} - \left\{\dfrac{1}{3}\right\}$

$(\forall x \in D_{f'}) : f'(x) = \dfrac{2(1-3x) - (2x+1)\cdot(-3)}{(1-3x)^2} = \dfrac{2-6x-(-6x-3)}{(1-3x)^2} = \dfrac{2-6x+6x+3}{(1-3x)^2} = \dfrac{5}{(1-3x)^2}$

d) $f(x) = \sin(2x^2) + \cos(2x+3)$

$D_f = D_{f'} = \mathbb{R}$

$(\forall x \in D_{f'}) : f'(x) = \cos(2x^2) \cdot 4x - \sin(2x+3) \cdot 2 = 4x\cos(2x^2) - 2\sin(2x+3)$

e) $f(x) = x^2 \cdot \sin(2x)$

$D_f = D_{f'} = \mathbb{R}$

$(\forall x \in D_{f'}) : f'(x) = 2x \cdot \sin(2x) + x^2 \cdot \cos(2x) \cdot 2 = 2x\sin(2x) + 2x^2\cos(2x)$

f) $f(x) = \sqrt{x^2+4x+3}$

condition: $x^2 + 4x + 3 \geq 0$

$\Delta = 16 - 12 = 4; x_1 = \dfrac{-4+2}{2} = -1; x_2 = \dfrac{-4-2}{2} = -3$

x	$-\infty$		-3		-1		$+\infty$
x^2+4x+3		$+$	0	$-$	0	$+$	

$D_f =]-\infty;-3] \cup [-1;+\infty[$ et $D_{f'} =]-\infty;-3[\cup]-1;+\infty[$

$(\forall x \in D_{f'}) : f'(x) = \dfrac{2x+4}{2\sqrt{x^2+4x+3}} = \dfrac{2(x+2)}{2\sqrt{x^2+4x+3}} = \dfrac{x+2}{\sqrt{x^2+4x+3}}$

g) $f(x) = \sin^4 x$

$D_f = D_{f'} = \mathbb{R}$

$(\forall x \in D_{f'}) : f'(x) = 4\sin^3 x \cos x$

h) $f(x) = \frac{3}{(2x+1)^4}$

condition: $(2x+1)^4 \neq 0 \Leftrightarrow 2x+1 \neq 0 \Leftrightarrow 2x \neq -1 \Leftrightarrow x \neq -\frac{1}{2}$

$D_f = D_{f'} = \mathbb{R} - \left\{-\frac{1}{2}\right\}$

$f(x) = 3(2x+1)^{-4}$

$(\forall x \in D_{f'})$: $f'(x) = 3 \cdot (-4) \cdot (2x+1)^{-5} \cdot 2 = -24 \cdot (2x+1)^{-5} = \frac{-24}{(2x+1)^5}$

i) $f(x) = \frac{x^2+3x-1}{x^2-4}$

condition: $x^2 - 4 \neq 0 \Leftrightarrow (x-2)(x+2) \neq 0 \Leftrightarrow x \neq 2$ et $x \neq -2$

$D_f = D_{f'} = \mathbb{R} - \{-2; 2\}$

$(\forall x \in D_{f'})$: $f'(x) = \frac{(2x+3)(x^2-4) - (x^2+3x-1)(2x)}{(x^2-4)^2} = \frac{(2x^3+3x^2-8x-12) - (2x^3+6x^2-2x)}{(x^2-4)^2}$

$= \frac{2x^3+3x^2-8x-12-2x^3-6x^2+2x}{(x^2-4)^2} = \frac{-3x^2-6x-12}{(x^2-4)^2}$

j) $f(x) = \sqrt{\sin x}$

condition: $\sin x \geq 0 \Leftrightarrow x \in [2k\pi; (2k+1)\pi], k \in \mathbb{Z}$

$D_f = [2k\pi; (2k+1)\pi], k \in \mathbb{Z}$ et $D_{f'} =]2k\pi; (2k+1)\pi[, k \in \mathbb{Z}$

$(\forall x \in D_{f'})$: $f'(x) = \frac{\cos x}{2\sqrt{\sin x}}$