Exercices sur la dérivation - étape 2 - corrigé

a) $f(x) = 2x^2 + x - 7$

$D_f = D_{f'} = \mathbb{R}$

$(\forall x \in D_{f'}) : f'(x) = 4x + 1$

b) $f(x) = 3x^3 + \frac{1}{x} - \cos(2x)$

condition: $x \neq 0$
$D_f = D_{f'} = \mathbb{R}^*$

$(\forall x \in D_{f'}) : f'(x) = 9x^2 - \frac{1}{x^2} - (-\sin(2x)) \cdot 2 = 9x^2 - \frac{1}{x^2} + 2\sin(2x)$

c) $f(x) = x \cdot \sqrt{x}$

condition: $x \geq 0$

$D_f = [0;+\infty[; D_{f'} =]0;+\infty[$

$(\forall x \in D_{f'}) : f'(x) = 1 \cdot \sqrt{x} + x \cdot \frac{1}{2\sqrt{x}} = \sqrt{x} + \frac{x}{2\sqrt{x}} = \sqrt{x} + \frac{\sqrt{x}}{2} = \frac{3\sqrt{x}}{2}$

d) $f(x) = \frac{(2x+1)^2}{1-4x}$

condition: $1 - 4x \neq 0 \Leftrightarrow 4x \neq 1 \Leftrightarrow x \neq \frac{1}{4}$

$D_f = D_{f'} = \mathbb{R} - \left\{\frac{1}{4}\right\}$

$(\forall x \in D_{f'}) : f'(x) = \frac{2(2x+1) \cdot 2 \cdot (1-4x) - (2x+1)^2(-4)}{(1-4x)^2} = \frac{4(2x+1)[(1-4x) + (2x+1)]}{(1-4x)^2}$
$\frac{(8x+4)(-2x+2)}{(1-4x)^2} = \frac{-16x^2 + 16x - 8x + 8}{(1-4x)^2} = \frac{-16x^2 + 8x + 8}{(1-4x)^2}$

e) $f(x) = (4x+1)^2(2-3x)^3$

$D_f = D_{f'} = \mathbb{R}$

$(\forall x \in D_{f'}) : f'(x) = 2(4x+1) \cdot 4 \cdot (2-3x)^3 + (4x+1)^2 \cdot 3(2-3x)^2 \cdot (-3) = (4x+1)(2-3x)^2[8(2-3x$
$= (4x+1)(2-3x)^2(16 - 24x - 36x - 9) = (4x+1)(2-3x)^2(-60x+7)$

f) $f(x) = \frac{x^2 + 3x - 1}{2x+1}$

condition: $2x + 1 \neq 0 \Leftrightarrow 2x \neq -1 \Leftrightarrow x \neq -\frac{1}{2}$

$D_f = D_{f'} = \mathbb{R} - \left\{-\frac{1}{2}\right\}$

$(\forall x \in D_{f'}) : f'(x) = \frac{(2x+3)(2x+1) - (x^2 + 3x - 1) \cdot 2}{(2x+1)^2} = \frac{4x^2 + 2x + 6x + 3 - 2x^2 - 6x + 2}{(2x+1)^2} = \frac{2x^2 + 2x + 5}{(2x+1)^2}$

g) $f(x) = \dfrac{4x}{\sqrt{x^2-9}}$

condition: $x^2 - 9 \geq 0$

$x^2 - 9 = 0 \Leftrightarrow (x-3)(x+3) = 0 \Leftrightarrow x = 3$ ou $x = -3$

x	$-\infty$		-3		3		$+\infty$
x^2+4x+3		$+$	0	$-$	0	$+$	

$D_f =]-\infty;-3] \cup [3;+\infty[$ et $D_{f'} =]-\infty;-3[\cup]3;+\infty[$

$$(\forall x \in D_{f'}) : f'(x) = \frac{4\sqrt{x^2-9} - 4x \cdot \frac{2x}{2\sqrt{x^2-9}}}{\left(\sqrt{x^2-9}\right)^2} = \frac{4\sqrt{x^2-9} - \frac{4x^2}{\sqrt{x^2-9}}}{x^2-9} = \frac{\frac{4(x^2-9)-4x^2}{\sqrt{x^2-9}}}{x^2-9} = \frac{4x^2-36-4x^2}{(x^2-9)\sqrt{x^2-9}} = \frac{-36}{(x^2-9)\sqrt{x^2-9}}$$

h) $f(x) = \dfrac{4x+1}{2x^2+5x-3}$

condition: $2x^2 + 5x + 3 \neq 0$

$\Delta = 25 - 24 = 1; x_1 = \dfrac{-5+1}{4} = -1; x_2 = \dfrac{-5-1}{4} = -\dfrac{3}{2}$

$D_f = D_{f'} = \mathbb{R} - \left\{-\dfrac{3}{2};-1\right\}$

$$(\forall x \in D_{f'}) : f'(x) = \frac{4(2x^2+5x-3) - (4x+1)(4x+5)}{(2x^2+5x-3)^2} = \frac{8x^2+20x-12-(16x^2+20x+4x+5)}{(2x^2+5x-3)^2}$$

$$= \frac{8x^2+20x-12-16x^2-24x-5}{(2x^2+5x-3)^2} = \frac{-8x^2-4x-17}{(2x^2+5x-3)^2}$$

i) $f(x) = x + \dfrac{2x}{x} - \dfrac{2}{9x^2}$

condition: $x \neq 0$

$D_f = D_{f'} = \mathbb{R}^*$

$f(x) = x + 2 - \dfrac{2}{9x}$

$$(\forall x \in D_{f'}) : f'(x) = 1 - \frac{2}{9} \cdot \left(-\frac{1}{x^2}\right) = 1 + \frac{2}{9x^2} = \frac{9x^2+2}{9x^2}$$

j) $f(x) = \sin x \cos x$

$D_f = D_{f'} = \mathbb{R}$

$$(\forall x \in D_{f'}) : f'(x) = \cos x \cdot \cos x + \sin x \cdot (-\sin x) = \cos^2 x - \sin^2 x$$