T2EE - Corrigé II,1 du 18.03.2011

Exercice 1 (**2** + **3** + **3** + **5** + **5** + **8** = **26 points**)

a) $f(x) = 12x^2 + 7x - 10$
$D_f = \mathbb{R}$

b) $f(x) = \dfrac{1-7x}{1-14x}$
condition: $1 - 14x \neq 0 \Leftrightarrow 14x \neq 1 \Leftrightarrow x \neq \frac{1}{14}$
$D_f = \mathbb{R} \setminus \left\{\frac{1}{14}\right\}$

c) $f(x) = \sqrt{5-3x} + \dfrac{4x}{9}$
condition: $5 - 3x \geq 0 \Leftrightarrow -3x \geq -5 \Leftrightarrow x \leq \frac{5}{3}$
$D_f = \left]-\infty; \frac{5}{3}\right]$

d) $f(x) = \sqrt{-4x^2 - 17x - 15}$
condition: $-4x^2 - 17x - 15 \geq 0$
$-4x^2 - 17x - 15 = 0 \overset{\Delta=49}{\Leftrightarrow} x = \dfrac{17-7}{-8} = -\dfrac{5}{4}$ ou $x = \dfrac{17+7}{-8} = -3$

x	$-\infty$		-3		$-\frac{5}{4}$		$+\infty$
$-4x^2 - 17x - 15$		$-$	0	$+$	0	$-$	

$D_f = \left[-3; -\frac{5}{4}\right]$

e) $f(x) = 2\sqrt{2x+4} - 3\sqrt{3-2x}$
conditions: $2x + 4 \geq 0 \Leftrightarrow x \geq -2$
$3 - 2x \geq 0 \Leftrightarrow -2x \geq -3 \Leftrightarrow x \leq \frac{3}{2}$
donc $D_f = \left[-2; \frac{3}{2}\right]$

f) $f(x) = \sqrt{\dfrac{2x+1}{3x-2}}$
condition: $\dfrac{2x+1}{3x-2} \geq 0$
$2x + 1 = 0 \Leftrightarrow x = -\frac{1}{2}$ / $3x - 2 = 0 \Leftrightarrow 3x = 2 \Leftrightarrow x = \frac{2}{3}$

x	$-\infty$		$-\frac{1}{2}$		$\frac{2}{3}$		$+\infty$
$2x+1$		$-$		$-$	0	$+$	
$3x-2$		$-$	0	$+$		$+$	
$\dfrac{2x+1}{3x-2}$		$+$	0	$-$	$\|$	$+$	

donc $D_f = \left]-\infty; -\frac{1}{2}\right] \cup \left]\frac{2}{3}; +\infty\right[$

Exercice 2 (**16 points**)

$f(x) = \dfrac{2x^2 - 3x - 7}{x^2 + x - 6}$
condition: $x^2 + x - 6 \neq 0$
$x^2 + x - 6 = 0 \overset{\Delta=25}{\Leftrightarrow} x = \dfrac{-1+5}{2} = 2$ ou $x = \dfrac{-1-5}{2} = -3$
$D_f = \mathbb{R} \setminus \{-3; 2\}$

▸ $\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty} \frac{2x^2}{x^2} = 2$ et $\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty} \frac{2x^2}{x^2} = 2$ A.H.: $y = 2$

▸ $\lim\limits_{x\to-3} \frac{\overbrace{2x^2-3x-7}^{\to 20}}{\underbrace{x^2+x-6}_{\to 0}}$ il faut calculer la limite en -3^+ et -3^-

x	$-\infty$		-3		2		$+\infty$
x^2+x-6		$+$	0	$-$	0	$+$	

$\lim\limits_{x\to-3^-} \frac{\overbrace{2x^2-3x-7}^{\to 20}}{\underbrace{x^2+x-6}_{\to 0^+}} = +\infty$ et $\lim\limits_{x\to-3^+} \frac{\overbrace{2x^2-3x-7}^{\to 20}}{\underbrace{x^2+x-6}_{\to 0^-}} = -\infty$ A.V.: $x = -3$

▸ $\lim\limits_{x\to 2} \frac{\overbrace{2x^2-3x-7}^{\to 20}}{\underbrace{x^2+x-6}_{\to 0}}$ il faut calculer la limite en 2^+ et 2^-

x	$-\infty$		-3		2		$+\infty$
x^2+x-6		$+$	0	$-$	0	$+$	

$\lim\limits_{x\to 2^-} \frac{\overbrace{2x^2-3x-7}^{\to -5}}{\underbrace{x^2+x-6}_{\to 0^-}} = +\infty$ et $\lim\limits_{x\to 2^+} \frac{\overbrace{2x^2-3x-7}^{\to -5}}{\underbrace{x^2+x-6}_{\to 0^+}} = -\infty$ A.V.: $x = 2$

Exercice 3 (2 + 2 + 8 = 12 points)

a) $\lim\limits_{x\to+\infty} \frac{3x^2-5x}{3x^3-7} = \lim\limits_{x\to+\infty} \frac{3x^2}{3x^3} = \lim\limits_{x\to+\infty} \frac{1}{x} = 0$

b) $\lim\limits_{x\to-\infty} \frac{4-5x^3}{4x^2+7x-5} = \lim\limits_{x\to-\infty} \frac{-5x^3}{4x^2} = \lim\limits_{x\to-\infty} \frac{-5x}{4} = +\infty$

c) $\lim\limits_{x\to 3} \frac{2x^2-5x-3}{3x^2-10x+3}$

$\lim\limits_{x\to 3} \frac{\overbrace{2x^2-5x-3}^{\to 0}}{\underbrace{3x^2-10x+3}_{\to 0}}$ forme indéterminée, il faut factoriser

$2x^2-5x-3 = 0 \overset{\Delta=49}{\Leftrightarrow} x = \frac{5+7}{4} = 3$ ou $x = \frac{5-7}{4} = -\frac{1}{2}$

donc: $2x^2-5x-3 = 2\left(x+\frac{1}{2}\right)(x-3) = (2x+1)(x-3)$

$3x^2-10x+3 = 0 \overset{\Delta=64}{\Leftrightarrow} x = \frac{10+8}{6} = 3$ ou $x = \frac{10-8}{6} = \frac{1}{3}$

donc: $3x^2-10x+3 = 3\left(x-\frac{1}{3}\right)(x-3) = (3x-1)(x-3)$

donc $\lim\limits_{x\to 3} f(x) = \lim\limits_{x\to 3} \frac{(2x+1)(x-3)}{(3x-1)(x-3)} = \lim\limits_{x\to 3} \frac{2x+1}{3x-1} = \frac{7}{8}$

Exercice 4 (6 points)

$\lim\limits_{x\to\pm\infty} [f(x)-(3x-7)] = \lim\limits_{x\to\pm\infty} \left[\frac{3x^2+8x-32}{x+5} - (3x-7)\right]$

$= \lim\limits_{x\to\pm\infty} \frac{3x^2+8x-32-(3x-7)(x+5)}{x+5} = \lim\limits_{x\to\pm\infty} \frac{3x^2+8x-32-(3x^2-7x+15x-35)}{x+5}$

$= \lim\limits_{x\to\pm\infty} \frac{3x^2+8x-32-3x^2-8x+35}{x+5} = \lim\limits_{x\to\pm\infty} \frac{3}{x+5} = \lim\limits_{x\to\pm\infty} \frac{3}{x} = 0$

Donc la droite d'équation $y = 3x-7$ est asymptote oblique à la courbe représentative de f.