T2EE - Corrigé du devoir en classe de mathématiques III,1

Exercice 1

a) $f(x) = 3x^4 - 2x^3 + \frac{5}{2}x^2 - x + 2\sqrt{3}$

$D_f = D_{f'} = \mathbb{R}$

$(\forall x \in D_{f'}) : f'(x) = 3 \cdot 4x^3 - 2 \cdot 3x^2 + \frac{5}{2} \cdot 2x - 1 = 12x^3 - 6x^2 + 5x - 1$

b) $f(x) = -5(7x^2 + 4)^4 + 7$

$D_f = D_{f'} = \mathbb{R}$

$(\forall x \in D_{f'}) : f'(x) = -5 \cdot 4(7x^2 + 4)^4 \cdot 7 \cdot 2x = -280x(7x^2 + 4)^3$

c) $f(x) = \frac{-3}{(5x+2)^2} = -3(5x+2)^{-2}$

condition: $5x + 2 \neq 0 \Leftrightarrow x \neq -\frac{2}{5}$

$D_f = D_{f'} = \mathbb{R} \setminus \left\{-\frac{2}{5}\right\}$

$(\forall x \in D_{f'}) : f'(x) = -3 \cdot (-2)(5x+2)^{-3} \cdot 5 = \frac{30}{(5x+2)^3}$

d) $f(x) = \frac{5 + 2x}{1 - 3x}$

condition: $1 - 3x \neq 0 \Leftrightarrow x \neq \frac{1}{3}$

$D_f = D_{f'} = \mathbb{R} \setminus \left\{\frac{1}{3}\right\}$

$(\forall x \in D_{f'}) : f'(x) = \frac{2(1-3x) - (5+2x)(-3)}{(1-3x)^2} = \frac{2 - 6x + 15 + 6x}{(1-3x)^2} = \frac{17}{(1-3x)^2}$

e) $f(x) = \overbrace{2x^2}^{u}\overbrace{\sqrt{x+1}}^{v}$

condition: $x + 1 \geq 0 \Leftrightarrow x \geq -1$

$D_f = [-1;+\infty[\qquad D_{f'} =]-1;+\infty[$

$(\forall x \in D_{f'}) : f'(x) = \overbrace{4x}^{u'}\overbrace{\sqrt{x+1}}^{v} + \overbrace{2x^2}^{u}\overbrace{\frac{1}{2\sqrt{x+1}}}^{v'} = 4x\sqrt{x+1} + \frac{x^2}{\sqrt{x+1}}$

$= \frac{4x(x+1) + x^2}{\sqrt{x+1}} = \frac{4x^2 + 4x + x^2}{\sqrt{x+1}} = \frac{5x^2 + 4x}{\sqrt{x+1}}$

f) $f(x)=\sqrt{x^2-4x-21}$

condition: $x^2-4x-21\geq 0$

$\Delta=16+84=100; x_1=\frac{4+10}{2}=7; x_2=\frac{4-10}{2}=-3$

x	$-\infty$		-3		7		$+\infty$
$x^2-4x-21$		$+$	0	$-$	0	$+$	

$D_f=]-\infty;-3]\cup[7;+\infty[$ et $D_{f'}=]-\infty;-3[\cup]7;+\infty[$

$(\forall x\in D_{f'}) : f'(x)=\frac{2x-4}{2\sqrt{x^2-4x-21}}=\frac{2(x-2)}{2\sqrt{x^2-4x-21}}=\frac{x-2}{\sqrt{x^2-4x-21}}$

g) $f(x)=5\sin^2 x-3\sin x$

$D_f=D_{f'}=\mathbb{R}$

$(\forall x\in D_{f'}) : f'(x)=5\cdot 2\sin x\cdot\cos x-3\cdot\cos x=10\sin x\cos x+3\cos x=\cos x(10\sin x+3)$

h) $f(x)=\frac{\overbrace{x^2-x}^{u}}{\underbrace{\sqrt{x}}_{v}}$

condition: $x>0$

$D_f=D_{f'}=]0;+\infty[$

$(\forall x\in D_{f'}) : f'(x)=\frac{\overbrace{(2x-1)}^{u'}\overbrace{\sqrt{x}}^{v}-\overbrace{(x^2-x)}^{u}\overbrace{\frac{1}{2\sqrt{x}}}^{v'}}{\underbrace{x}_{v^2}}=\frac{\frac{2(2x-1)x-(x^2-x)}{2\sqrt{x}}}{x}=\frac{4x^2-2x-x^2+x}{2x\sqrt{x}}=\frac{3x^2-x}{2x\sqrt{x}}=$

i) $f(x)=\overbrace{(3x+4)^3}^{u}\overbrace{(5-4x)^4}^{v}$

$D_f=D_{f'}=\mathbb{R}$

$(\forall x\in D_{f'}) : f'(x)=\overbrace{3(3x+4)^2\cdot 3}^{u'}\cdot\overbrace{(5-4x)^4}^{v}+\overbrace{(3x+4)^3}^{u}\cdot\overbrace{4(5-4x)^3\cdot(-4)}^{v'}$

$=9(3x+4)^2(5-4x)^4-16(3x+4)^3(5-4x)^3$

$=(3x+4)^2(5-4x)^3[9(5-4x)-16(3x+4)]$

$=(3x+4)^2(5-4x)^3(45-36x-48x-64)$

$=(3x+4)^2(5-4x)^3(-84x-19)$

j) $f(x)=-3\cos^4(-5x+6)$

$D_f=D_{f'}=\mathbb{R}$

$(\forall x\in D_{f'}) : f'(x)=-3\cdot 4\cdot\cos^3(-5x+6)\cdot(-\sin(-5x+6))\cdot(-5)=-60\cos^3(-5x+6)\sin(-5x+6)$