

T2EE - Corrigé du devoir en classe de mathématiques I,1

Exercice 1

$$a) z_1 = 3i^3 + 2i^2 + i = -3 - 2i + i = -2 - 2i$$

$$b) z_2 = (5 + 4i)(2 - i) - i(4 - 4i) = 10 - 5i + 8i + 4 - 4i - 4 = 10 - i$$

$$c) z_3 = (1 - 3i)^3 = (1 - 3i)^2(1 - 3i) = (1 - 6i - 9)(1 - 3i) = (1 - 3i)(-8 - 6i) = -8 - 6i + 24i - 18$$

$$d) z_4 = \frac{-7 - 2i}{i^2} = \frac{-7 - 2i}{-1} = 7 + 2i$$

$$e) z_5 = \frac{2 + 3i}{1 + i} \cdot \frac{1 - i}{1 - i} = \frac{2 - 2i + 3i + 3}{1 + 1} = \frac{5 + i}{2} = \frac{5}{2} + \frac{1}{2}i$$

$$f) z_6 = \sqrt{8} \left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right) = \sqrt{8} \left(\frac{\sqrt{2}}{2} - i \frac{\sqrt{2}}{2} \right) = \frac{\sqrt{16}}{2} - i \frac{\sqrt{16}}{2} = -2 + 2i$$

Exercice 2

$$a) z_1 = -5 - 5i ; a = -5, b = -5$$

$$r = \sqrt{a^2 + b^2} = \sqrt{25 + 25} = \sqrt{50} = 5\sqrt{2}$$

$$\cos \theta = \frac{a}{r} = \frac{-5}{5\sqrt{2}} = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}; \sin \theta = \frac{b}{r} = \frac{-5}{5\sqrt{2}} = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

$$\text{donc } \theta = \frac{5\pi}{4} \left(\text{ou } -\frac{3\pi}{4} \right) \text{ et } z_1 = 5\sqrt{2} \left(\cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4} \right)$$

$$b) z_2 = 14i ; a = 0, b = 14$$

$$r = \sqrt{a^2 + b^2} = \sqrt{0 + 14^2} = 14$$

$$\cos \theta = \frac{a}{r} = \frac{0}{14} = 0; \sin \theta = \frac{b}{r} = \frac{14}{14} = 1$$

$$\text{donc } \theta = \frac{\pi}{2} \text{ et } z_2 = 14 \left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right)$$

$$c) z_3 = -3 \left(\cos \frac{5\pi}{6} - i \sin \frac{5\pi}{6} \right) = 3 \left(-\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} \right) = 3 \left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right)$$

Exercice 3

$$a) -2z + 2(i - 3) = 5i + 9$$

$$\Leftrightarrow -2z + 2i - 6 = 5i + 9$$

$$\Leftrightarrow -2z = 15 + 3i \quad | : (-2)$$

$$\Leftrightarrow z = \frac{15 + 3i}{-2} = -\frac{15}{2} - \frac{3}{2}i \quad S = \left\{ -\frac{15}{2} - \frac{3}{2}i \right\}$$

$$b) iz + 7i - 5 = 4(2 + 3i)$$

$$\Leftrightarrow iz + 7i - 5 = 8 + 12i$$

$$\Leftrightarrow iz = 13 + 5i \quad | : i$$

$$\Leftrightarrow z = \frac{13 + 5i}{i} \cdot \frac{-i}{-i}$$

$$\Leftrightarrow z = \frac{-13i + 5}{1} = 5 - 13i \quad S = \{5 - 13i\}$$

$$c) (3 + i)z = 2 - 4i$$

$$\Leftrightarrow z = \frac{2 - 4i}{3 + i} \cdot \frac{3 - i}{3 - i}$$

$$\Leftrightarrow z = \frac{6 - 2i - 12i - 4}{9 + 1} = \frac{2 - 14i}{10} = \frac{1}{5} - \frac{7}{5}i \quad S = \left\{ \frac{1}{5} - \frac{7}{5}i \right\}$$

Exercice 4

$$a) z = 2\sqrt{3} + i\sqrt{13}, |z| = \sqrt{(2\sqrt{3})^2 + (\sqrt{13})^2} = \sqrt{12 + 13} = \sqrt{25} = 5$$

$$b) z = i\sqrt{7} + \sqrt{2}, \bar{z} = -i\sqrt{7} + \sqrt{2}$$

$$c) z = 3 + \sqrt{2}, \bar{z} = 3 + \sqrt{2}$$

$$d) \text{ Si } z = -\sqrt{3} + i, r = \sqrt{(-\sqrt{3})^2 + 1^2} = \sqrt{3 + 1} = \sqrt{4} = 2$$

$$\cos \theta = \frac{a}{r} = \frac{-\sqrt{3}}{2}; \sin \theta = \frac{b}{r} = \frac{1}{2}$$

$$\text{Donc } \arg(z) = \theta = \frac{5\pi}{6}$$