T2EE - Corrigé du devoir en classe de mathématiques I,2

Exercice 1

a) $z_1 = \left(\sqrt{3} + \sqrt{6}i\right)\left(-1 + \sqrt{2}i\right) = -\sqrt{3} + \sqrt{6}i - \sqrt{6}i - \sqrt{12} = -\sqrt{3} - 2\sqrt{3} = \boxed{-3\sqrt{3}}$

b) $z_2 = \frac{3-4i}{5+2i} \cdot \frac{5-2i}{5-2i} = \frac{15-6i-20i-8}{25+4} = \boxed{\frac{7}{29} - \frac{26}{29}i}$

c) $z_3 = \left(-1 - \sqrt{3}i\right)^9$; $r = \sqrt{1+3} = 2$; $\cos\theta = \frac{a}{r} = -\frac{1}{2}$; $\sin\theta = \frac{b}{r} = -\frac{\sqrt{3}}{2}$ donc $\theta = -\frac{2\pi}{3}$.

donc

$\left[2\left(\cos\left(-\frac{2\pi}{3}\right) + i\sin\left(-\frac{2\pi}{3}\right)\right)\right]^{11} = 2^9\left(\cos\left(-\frac{18\pi}{3}\right) + i\sin\left(-\frac{18\pi}{3}\right)\right) = 512(\cos(-6\pi) + i\sin(-6\pi)) = \boxed{512}$

d) $z_4 = e^{\frac{\pi}{7}i} \cdot e^{\frac{2\pi}{7}i} \cdot e^{\frac{4\pi}{7}i} = e^{\frac{7\pi}{7}i} = e^{\pi i} = \boxed{-1}$

e) $z_5 = \left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right) = \cos\left(\frac{3\pi}{4} - \frac{\pi}{4}\right) + i\sin\left(\frac{3\pi}{4} - \frac{\pi}{4}\right)$

$= \cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right) = \boxed{i}$

f) $z_6 = \frac{4e^{\frac{5\pi}{12}i}}{2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)} = \frac{4e^{\frac{5\pi}{12}i}}{2e^{\frac{\pi}{6}i}} = 2e^{\left(\frac{5\pi}{12} - \frac{\pi}{6}\right)i} = 2e^{\frac{3\pi}{12}i} = 2e^{\frac{\pi}{4}i} = \boxed{\sqrt{2} + \sqrt{2}i}$

Exercice 2

a) $z_1 \cdot z_2 = (-2+2i)\left(\sqrt{3} - i\right) = -2\sqrt{3} + 2i + 2i\sqrt{3} + 2 = \boxed{\left(2 - 2\sqrt{3}\right) + \left(2 + 2\sqrt{3}\right)i}$

b) $z_1 = -2 + 2i$; $a = -2, b = 2$; $r = \sqrt{a^2 + b^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

$$\left.\begin{array}{l} \cos\theta = \frac{a}{r} = \frac{-2}{2\sqrt{2}} = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2} \\ \sin\theta = \frac{b}{r} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \end{array}\right\} \text{donc } \theta = \frac{3\pi}{4} \text{ et } \boxed{z_1 = 2\sqrt{2}e^{\frac{3\pi}{4}i}}$$

$z_2 = \sqrt{3} - i$; ;$a = \sqrt{3}, b = -1$; $r = \sqrt{a^2 + b^2} = \sqrt{3+1} = 2$

$$\left.\begin{array}{l} \cos\theta = \frac{a}{r} = \frac{\sqrt{3}}{2} \\ \sin\theta = \frac{b}{r} = -\frac{1}{2} \end{array}\right\} \text{donc } \theta = -\frac{\pi}{6} \text{ et } \boxed{z_2 = 2e^{-\frac{\pi}{6}i}}$$

c) $z_1 \cdot z_2 = 4\sqrt{2}e^{\left(\frac{3\pi}{4} - \frac{\pi}{6}\right)i} = \boxed{4\sqrt{2}e^{\frac{7\pi}{12}i}}$

d) $4\sqrt{2}\left(\cos\left(\frac{7\pi}{12}\right) + i\sin\left(\frac{7\pi}{12}\right)\right) = \left(2 - 2\sqrt{3}\right) + \left(2 + 2\sqrt{3}\right)i$

$$\text{donc } \left\{\begin{array}{l} 4\sqrt{2}\cos\left(\frac{7\pi}{12}\right) = 2 - 2\sqrt{3} \Leftrightarrow \cos\left(\frac{7\pi}{12}\right) = \frac{2-2\sqrt{3}}{4\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2} - 2\sqrt{6}}{4 \cdot 2} = \boxed{\frac{\sqrt{2} - \sqrt{6}}{4}} \\ 4\sqrt{2}\sin\left(\frac{7\pi}{12}\right) = 2 + 2\sqrt{3} \Leftrightarrow \sin\left(\frac{7\pi}{12}\right) = \frac{2+2\sqrt{3}}{4\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2} + 2\sqrt{6}}{4 \cdot 2} = \boxed{\frac{\sqrt{2} + \sqrt{6}}{4}} \end{array}\right.$$

Exercice 3

a) $3z - (2 + 4i) = 4(2 + 3i) + 5z$

$\Leftrightarrow\ 3z - 2 - 4i = 8 + 12i + 5z$

$\Leftrightarrow\ -2z = 10 + 16i \quad |:(-2)$

$\Leftrightarrow\ z = -5 - 8i \qquad \boxed{S = \{-5 - 8i\}}$

b) $3 + iz = 4i + z$

$\Leftrightarrow\ iz - z = -3 + 4i$

$\Leftrightarrow\ (-1 + i)z = -3 + 4i \quad |:(-1 + i)$

$\Leftrightarrow\ z = \dfrac{-3 + 4i}{-1 + i} \cdot \dfrac{-1 - i}{-1 - i} = \dfrac{3 + 3i - 4i + 4}{1 + 1} = \dfrac{7}{2} - \dfrac{1}{2}i \qquad \boxed{S = \left\{\dfrac{7}{2} - \dfrac{1}{2}i\right\}}$

c) $2z \cdot e^{\frac{14\pi}{15}i} - 4e^{\frac{4\pi}{15}i} = 0$

$\Leftrightarrow\ 2z \cdot e^{\frac{14\pi}{15}i} = 4e^{\frac{4\pi}{15}i} \quad |:\left(2e^{\frac{14\pi}{15}i}\right)$

$\Leftrightarrow\ z = \dfrac{4e^{\frac{4\pi}{15}i}}{2e^{\frac{14\pi}{15}i}} = 2e^{\left(\frac{4\pi}{15} - \frac{14\pi}{15}\right)i} = 2e^{-\frac{2\pi}{3}i} = 2\left(\cos\left(-\dfrac{2\pi}{3}\right) + i\sin\left(-\dfrac{2\pi}{3}\right)\right)$

$\Leftrightarrow\ z = 2\left(-\dfrac{1}{2} - \dfrac{\sqrt{3}}{2}\right) = -1 - \sqrt{3}i \qquad \boxed{S = \left\{-1 - \sqrt{3}i\right\}}$

d) $z^2 + 2iz - 5 = 0$

$a = 1; b = 2i; c = -5 : \Delta = (2i)^2 - 4 \cdot (-5) = -4 + 20 = 16$

$z_1 = \dfrac{-2i + 4}{2} = 2 - i$ et $z_2 = \dfrac{-2i - 4}{2} = -2 - i \qquad \boxed{S = \{2 - i; -2 - i\}}$