T2EE - Corrigé du devoir en classe de mathématiques II,2

Exercice 1

a) $f(x) = 3;\ D_f = D_{f'} = \mathbb{R}$
$(\forall x \in D_{f'}) : f'(x) = 0$

b) $f(x) = -2x + 5;\ D_f = D_{f'} = \mathbb{R}$
$(\forall x \in D_{f'}) : f'(x) = -2$

c) $f(x) = 4x^3 - 5x^2 + 2x - \sqrt{2};\ D_f = D_{f'} = \mathbb{R}$
$(\forall x \in D_{f'}) : f'(x) = 12x^2 - 10x + 2$

d) $f(x) = \dfrac{1}{3x+5}$; cond: $3x + 5 \neq 0 \Leftrightarrow x \neq -\frac{5}{3}; D_f = D_{f'} = \mathbb{R} - \left\{-\frac{5}{3}\right\}$
$(\forall x \in D_{f'}) : f'(x) = -\dfrac{3}{(3x+5)^2}$

e) $f(x) = \sqrt{4x-2}$; cond: $4x - 2 \geq 0 \Leftrightarrow x \geq \frac{1}{2}; D_f = \left[\frac{1}{2};+\infty\right[$ et $D_{f'} = \left]\frac{1}{2};+\infty\right[$
$(\forall x \in D_{f'}) : f'(x) = \dfrac{4}{2\sqrt{4x-2}} = \dfrac{2}{\sqrt{4x-2}}$

f) $f(x) = x^2\sqrt{x}$; cond: $x \geq 0; D_f = [0;+\infty[$ et $D_{f'} =]0;+\infty[$
$(\forall x \in D_{f'}) : f'(x) = 2x\sqrt{x} + x^2 \cdot \dfrac{1}{2\sqrt{x}} = \dfrac{2x\sqrt{x} \cdot 2\sqrt{x} + x^2}{2\sqrt{x}} = \dfrac{4x^2 + x^2}{2\sqrt{x}} = \dfrac{5x^2}{2\sqrt{x}}$

g) $f(x) = \dfrac{2x-7}{x^2+3x}$; cond: $x^2 + 3x \neq 0 \Leftrightarrow x(x+3) \neq 0 \Leftrightarrow x \neq 0$ et $x \neq -3$; $D_f = D_{f'} = \mathbb{R} - \{0;-3\}$
$(\forall x \in D_{f'}) : f'(x) = \dfrac{2(x^2+3x) - (2x-7)(2x+3)}{(x^2+3x)^2} = \dfrac{2x^2 + 6x - (4x^2 + 6x - 14x - 21)}{(x^2+3x)^2}$
$= \dfrac{2x^2 + 6x - 4x^2 + 8x + 21}{(x^2+3x)^2} = \dfrac{-2x^2 + 14x + 21}{(x^2+3x)^2}$

Exercice 2

$\lim\limits_{x\to\pm\infty}\left[\dfrac{x^2-5x-11}{x+2} - (x-7)\right] = \lim\limits_{x\to\pm\infty}\dfrac{x^2-5x-11-(x-7)(x+2)}{x+2} = \lim\limits_{x\to\pm\infty}\dfrac{x^2-5x-11-(x^2-7x+2x-14)}{x+2}$
$= \lim\limits_{x\to\pm\infty}\dfrac{x^2-5x-11-x^2+5x+14}{x+2} = \lim\limits_{x\to\pm\infty}\dfrac{3}{x+2} = 0$
Donc la droite d'équation $y = x - 7$ est asymptote oblique à la courbe d'équation $y = \dfrac{x^2-5x-11}{x+2}$.

Exercice 3

$f : x \mapsto \dfrac{x^2 - x - 6}{-x^2 + 3x + 10}$

a) condition: $-x^2 + 3x + 10 \neq 0 \overset{\Delta=49}{\Leftrightarrow} x \neq \dfrac{-3+7}{-2}$ et $x \neq \dfrac{-3-7}{-2} \Leftrightarrow x \neq -2$ et $x \neq 5$.

Donc $D_f = \mathbb{R} \setminus \{-2; 5\}$.

b) ▹ $\lim\limits_{x \to -\infty} f(x) = \lim\limits_{x \to -\infty} \dfrac{x^2}{-x^2} = -1$ A.H.:$y = -1$

▹ de même: $\lim\limits_{x \to +\infty} f(x) = -1$ A.H.:$y = -1$

▹ $\lim\limits_{x \to -2} \dfrac{\overbrace{x^2 - x - 6}^{\to 0}}{\underbrace{-x^2 + 3x + 10}_{\to 0}}$ f.i." $\dfrac{0}{0}$ ", il faut factoriser.

$x^2 - x - 6 = 0 \overset{\Delta=25}{\Leftrightarrow} x = -2$ ou $x = 3$

Ainsi: $\lim\limits_{x \to -2} \dfrac{x^2 - x - 6}{-2x^2 + 6x + 20} = \lim\limits_{x \to -2} \dfrac{(x+2)(x-3)}{-(x+2)(x-5)} = \lim\limits_{x \to -2} \dfrac{(x-3)}{-(x-5)} = \dfrac{-5}{-(-7)} = -\dfrac{5}{7}$

▹ $\lim\limits_{x \to 5} \dfrac{\overbrace{x^2 - x - 6}^{\to 14}}{\underbrace{-2x^2 + 6x + 20}_{\to 0}}$ il faut distinguer 5^+ et 5^-.

x	$-\infty$		-2		5		$+\infty$
$-x^2 + 3x + 10$		$-$	0	$+$	0	$-$	

$\lim\limits_{x \to 5^-} \dfrac{\overbrace{x^2 - x - 6}^{\to 14}}{\underbrace{-x^2 + 3x + 10}_{\to 0^+}} = +\infty$ et $\lim\limits_{x \to 5^+} \dfrac{\overbrace{x^2 - x - 6}^{\to 14}}{\underbrace{-x^2 + 3x + 10}_{\to 0^-}} = -\infty$ A.V.:$x = 5$

c) $(\forall x \in D_{f'}) : f'(x) = \dfrac{(2x-1)(-x^2+3x+10) - (x^2-x-6)(-2x+3)}{(-x^2+3x+10)^2}$

$= \dfrac{-2x^3 + 6x^2 + 20x + x^2 - 3x - 10 - (-2x^3 + 3x^2 + 2x^2 - 3x + 12x - 18)}{(-x^2+3x+10)^2}$

$= \dfrac{-2x^3 + 7x^2 + 17x - 10 + 2x^3 - 5x^2 - 9x + 18}{(-x^2+3x+10)^2} =$

$= \dfrac{2x^2 + 8x + 8}{(-x^2+3x+10)^2}$

$\left[= \dfrac{2(x^2+4x+4)}{[-(x-5)(x+2)]^2} = \dfrac{2(x+2)^2}{(x-5)^2(x+2)^2} = \dfrac{2}{(x-5)^2} \right]$