T2EE - Corrigé du devoir en classe de mathématiques III,1

Exercice 1

a) $f(x) = x^3 + 2x^2 - 3x + 5 - \frac{4}{x}$

condition: $x \neq 0$

$D_f = D_{f'} = \mathbb{R}^*$

$(\forall x \in D_{f'}) : f'(x) = 3x^2 + 4x - 3 + \frac{4}{x^2}$

b) $f(x) = -2(x^2 + 4)^6 + 1$

$D_f = D_{f'} = \mathbb{R}$

$(\forall x \in D_{f'}) : f'(x) = -2 \cdot 6(x^2 + 4)^5 \cdot 2x = -24x(x^2 + 4)^5$

c) $f(x) = \cos(2x) + \sin\left(\frac{x}{4} + 1\right) - \tan\left(\frac{\pi}{4}\right)$

$D_f = D_{f'} = \mathbb{R}$

$(\forall x \in D_{f'}) : f'(x) = -\sin(2x) \cdot 2 + \cos\left(\frac{x}{4} + 1\right) \cdot \frac{1}{4} = -2\sin(2x) + \frac{1}{4}\cos\left(\frac{x}{4} + 1\right)$

d) $f(x) = \frac{-4}{x^2 + 5x + 6}$

condition: $x^2 + 5x + 6 \neq 0$

$\Delta = 25 - 24 = 1; x \neq \frac{-5+1}{2} = -2$ et $x \neq \frac{-5-1}{2} = -3$

$D_f = D_{f'} = \mathbb{R} - \{-2; -3\}$

$(\forall x \in D_{f'}) : f'(x) = -4 \cdot \left(-\frac{2x + 5}{(x^2 + 5x + 6)^2}\right) = \frac{4(2x + 5)}{(x^2 + 5x + 6)^2}$

e) $f(x) = x^2 \cdot \sin(x^2) - x^2$

$D_f = D_{f'} = \mathbb{R}$

$(\forall x \in D_{f'}) : f'(x) = 2x \cdot \sin(x^2) + x^2\cos(x^2) \cdot 2x - 2x = 2x\sin(x^2) + 2x^3\cos(2x) - 2x$

f) $f(x) = \sqrt{-2x^2 - 7x - 3}$

condition: $-2x^2 - 7x - 3 \geq 0$

$\Delta = 49 - 24 = 25; x_1 = \frac{7+5}{-4} = -3; x_2 = \frac{7-5}{-4} = -\frac{1}{2}$

x	$-\infty$		-3		$-\frac{1}{2}$		$+\infty$
$-2x^2 - 7x - 3$		$-$	0	$+$	0	$-$	

$D_f = \left[-3; -\frac{1}{2}\right]$ et $D_{f'} = \left]-3; -\frac{1}{2}\right[$

$(\forall x \in D_{f'}) : f'(x) = \frac{-4x - 7}{2\sqrt{-2x^2 - 7x - 3}}$

g) $f(x) = \frac{5-2x}{3+4x}$

condition: $3+4x \neq 0 \Leftrightarrow 4x \neq -3 \Leftrightarrow x \neq -\frac{3}{4}$

$D_f = D_{f'} = \mathbb{R} - \left\{-\frac{3}{4}\right\}$

$$(\forall x \in D_{f'}) : f'(x) = \frac{-2(3+4x)-(5-2x)\cdot 4}{(3+4x)^2} = \frac{-6-8x-20+8x}{(3+4x)^2} = \frac{-26}{(3+4x)^2}$$

h) $f(x) = \frac{x^2+2x-1}{x^2-4x}$

condition: $x^2-4x \neq 0 \Leftrightarrow x(x-4) \neq 0 \Leftrightarrow x \neq 0$ et $x \neq 4$

$D_f = D_{f'} = \mathbb{R} \setminus \{0;4\}$

$$(\forall x \in D_{f'}) : f'(x) = \frac{(2x+2)(x^2-4x)-(x^2+2x-1)(2x-4)}{(x^2-4x)^2}$$

$$= \frac{2x^3-8x^2+2x^2-8x-2x^3+4x^2-4x^2+8x+2x-4}{(x^2-4x)^2} = \frac{-6x^2+2x-4}{(x^2-4x)^2}$$

i) $f(x) = \frac{3x^2-4x}{(x-1)^2}$

condition: $(x-1)^2 \neq 0 \Leftrightarrow x-1 \neq 0 \Leftrightarrow x \neq 1$

$D_f = D_{f'} = \mathbb{R} - \{1\}$

$$(\forall x \in D_{f'}) : f'(x) = \frac{(6x-4)(x-1)^2-(3x^2-4x)\cdot 2(x-1)}{(x-1)^4} = \frac{(x-1)[(6x-4)(x-1)-2(3x^2-4x)]}{(x-1)^4}$$

$$= \frac{6x^2-4x-6x+4-6x^2+8x}{(x-1)^3} = \frac{4-2x}{(x-1)^3}$$

j) $f(x) = \frac{x^2-x}{\sqrt{x}}$

condition: $x > 0$

$D_f = D_{f'} =]0;+\infty[$

$$(\forall x \in D_{f'}) : f'(x) = \frac{(2x-1)\sqrt{x}-(x^2-x)\frac{1}{2\sqrt{x}}}{x} = \frac{\frac{2(2x-1)x-(x^2-x)}{2\sqrt{x}}}{x} = \frac{4x^2-2x-x^2+x}{2x\sqrt{x}} = \frac{3x^2-x}{2x\sqrt{x}} = \frac{3x-1}{2\sqrt{x}}$$