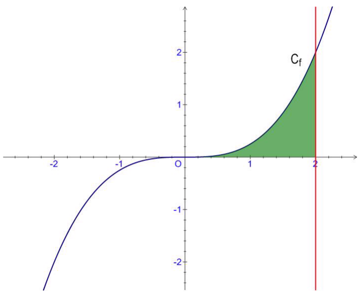
Aide-Mémoire T3EE - INTÉGRATION: CALCUL D'AIRES

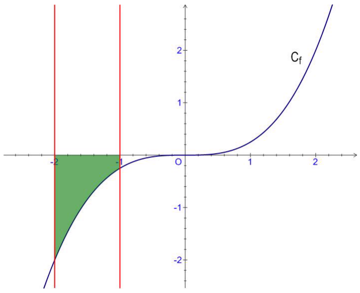
Aire comprise entre l'axe des abscisses, la courbe C_f et les droites d'équations $x = a$ et $x = b$

-si $\mathbf{f}(x) \geq \mathbf{0}$ entre a et b : $\mathcal{A} = \int_a^b f(x)dx$

-si $\mathbf{f}(x) \leq \mathbf{0}$ entre a et b : $\mathcal{A} = -\int_a^b f(x)dx$

Exemples:





$f(x) = \frac{1}{4}x^3, a = 0$ et $b = 2$

$\mathcal{A} = \int_0^2 \frac{1}{4}x^3dx = \left[\frac{1}{4} \cdot \frac{x^4}{4}\right]_0^2$

$= \frac{2^4}{16} = \frac{16}{16} = 1$ u.a.

$f(x) = \frac{1}{4}x^3, a = -2$ et $b = -1$

$\mathcal{A} = -\int_{-2}^{-1} \frac{1}{4}x^3dx = -\left[\frac{1}{4} \cdot \frac{x^4}{4}\right]_{-2}^{-1}$

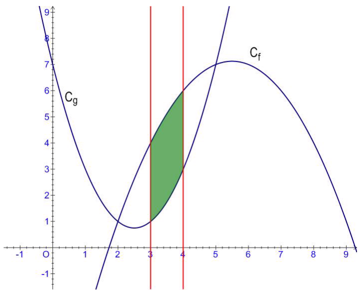
$= -\frac{(-1)^4}{16} + \frac{(-2)^4}{16} = -\frac{1}{16} + \frac{16}{16} = \frac{15}{16}$ u.a.

Aire comprise entre les courbes C_f et C_g et les droites d'équations $x = a$ et $x = b$

-si $f(x) \geq g(x)$ entre a et b : $\mathcal{A} = \int_a^b [f(x) - g(x)]dx$

-si $g(x) \geq f(x)$ entre a et b : $\mathcal{A} = \int_a^b [g(x) - f(x)]dx$

Exemple:



$f(x) = -\frac{1}{2}x^2 + \frac{11}{2}x - 8$ et $g(x) = x^2 - 5x + 7$

$\mathcal{A} = \int_3^4 [f(x) - g(x)]dx = \int_3^4 \left[\left(-\frac{1}{2}x^2 + \frac{11}{2}x - 8\right) - (x^2 - 5x + 7)\right]dx$

$= \int_3^4 \left(-\frac{3}{2}x^2 + \frac{21}{2}x - 15\right)dx = \left[-\frac{1}{2}x^3 + \frac{21}{4}x^2 - 15x\right]_3^4 = \frac{13}{4}$ u.a.

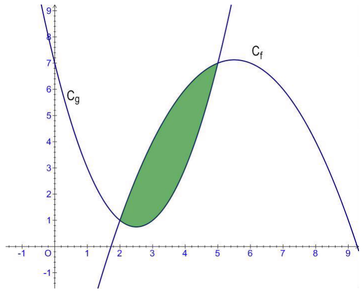
Aire comprise entre les courbes C_f et C_g

On détermine les abscisses des points d'intersection a et b en posant $f(x) = g(x)$.

-si $f(x) \geq g(x)$ entre a et b : $\mathcal{A} = \int_a^b [f(x) - g(x)]dx$

-si $g(x) \geq f(x)$ entre a et b : $\mathcal{A} = \int_a^b [g(x) - f(x)]dx$

Exemples:



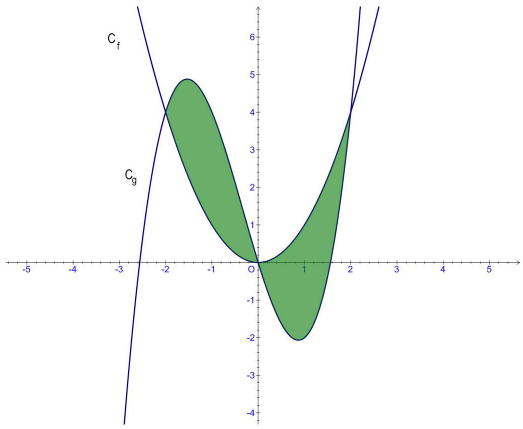
$f(x) = -\frac{1}{2}x^2 + \frac{11}{2}x - 8$ et $g(x) = x^2 - 5x + 7$

points d'intersection: $f(x) = g(x) \Leftrightarrow -\frac{1}{2}x^2 + \frac{11}{2}x - 8 = x^2 - 5x + 7$

$\Leftrightarrow \frac{3}{2}x^2 - \frac{21}{2}x + 15 = 0 \Leftrightarrow 3x^2 - 21x + 30 = 0 \overset{\Delta=81}{\Leftrightarrow} x = 2$ ou $x = 5$

$\mathcal{A} = \int_2^5 [f(x) - g(x)]dx = \int_2^5 \left[\left(-\frac{1}{2}x^2 + \frac{11}{2}x - 8\right) - (x^2 - 5x + 7)\right]dx$

$= \int_2^5 \left(-\frac{3}{2}x^2 + \frac{21}{2}x - 15\right)dx = \left[-\frac{1}{2}x^3 + \frac{21}{4}x^2 - 15x\right]_2^5 = \frac{27}{4}$ u. a.



$f(x) = x^2$ et $g(x) = x^3 + x^2 - 4x$

points d'intersection: $f(x) = g(x) \Leftrightarrow x^2 = x^3 + x^2 - 4x \Leftrightarrow x^3 - 4x = 0$

$\Leftrightarrow x(x^2 - 4) = 0 \Leftrightarrow x(x-2)(x+2) = 0 \Leftrightarrow x = 0$ ou $x = -2$ ou $x = 2$.

$\mathcal{A} = \int_{-2}^0 [g(x) - f(x)]dx + \int_0^2 [f(x) - g(x)]dx = \int_{-2}^0 [x^3 + x^2 - 4x - x^2]dx + \int_0^2 [x^2 - (x^3 + x^2 - 4x)]dx$

$= \int_{-2}^0 (x^3 - 4x)dx + \int_0^2 (-x^3 + 4x)dx = \left[\frac{x^4}{4} - 4 \cdot \frac{x^2}{2}\right]_{-2}^0 + \left[-\frac{x^4}{4} + 4 \cdot \frac{x^2}{2}\right]_0^2$

$= 0 - \left(\frac{16}{4} - \frac{16}{2}\right) + \left(-\frac{16}{4} + \frac{16}{2}\right) = -4 + 8 - 4 + 8 = 8$ u.a.