EXAMEN 2006 - T3EC/T3EE - Corrigé

Exercice 1

$f(x) = \dfrac{4x-1}{(x-2)^2}$

condition: $(x-2)^2 \neq 0 \Leftrightarrow x-2 \neq 0 \Leftrightarrow x \neq 2$
donc $D_f = \mathbb{R} \setminus \{2\}$

▸ $(\forall x \in \mathbb{R} \setminus \{2\}) : f'(x) = \dfrac{4 \cdot (x-2)^2 - (4x-1) \cdot 2(x-2)}{(x-2)^4} = \dfrac{(x-2)[4(x-2) - 2(4x-1)]}{(x-2)^4}$
$= \dfrac{4x-8-(8x-2)}{(x-2)^3} = \dfrac{4x-8-8x+2}{(x-2)^3} = \dfrac{-4x-6}{(x-2)^3}$

▸ $f'(x) = 0 \Leftrightarrow -4x-6 = 0 \Leftrightarrow 4x = -6 \Leftrightarrow x = -\dfrac{6}{4} = -\dfrac{3}{2}$

x	$-\infty$		$-\frac{3}{2}$		2		$+\infty$
$-4x-6$		$+$	0	$-$		$-$	
$(x-2)^3$		$-$		$-$	0	$+$	
$f'(x)$		$-$	0	$+$	$\|$	$-$	

▸ $f(x) = \dfrac{4x-1}{(x-2)^2} = \dfrac{4x-1}{x^2-4x+4}$

▸ $\left.\begin{array}{l} \lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty} \dfrac{4x}{x^2} = \lim\limits_{x\to+\infty} \dfrac{4}{x} = 0 \\ \text{de même : } \lim\limits_{x\to-\infty} f(x) = 0 \end{array}\right\}$ A.H.: $y = 0$

▸ $\lim\limits_{x\to 2} \dfrac{\overbrace{4x-1}^{\to 7}}{\underbrace{(x-2)^2}_{\to 0^+}} = +\infty$ A.V.: $x = 2$

▸ $f\left(-\dfrac{3}{2}\right) = \dfrac{4 \cdot \left(-\frac{3}{2}\right) - 1}{\left(-\frac{3}{2} - 2\right)^2} = -\dfrac{4}{7}$

x	$-\infty$		$-\frac{3}{2}$		2		$+\infty$
f'		$-$	0	$+$	$\|$	$-$	
f	0	$\searrow$	$-\frac{4}{7}$	$\nearrow$	$+\infty \ \| \ +\infty$	$\searrow$	0

coordonnées des points d'intersection de C_f avec les axes:

▸ $C_f \cap (Ox)$:
$f(x) = 0 \Leftrightarrow 4x-1 = 0 \Leftrightarrow 4x = 1 \Leftrightarrow x = \dfrac{1}{4}$
donc $C_f \cap (Ox) = \left\{\left(\dfrac{1}{4};0\right)\right\}$

▸ $C_f \cap (Oy)$:
$f(0) = \dfrac{-1}{4} = -\dfrac{1}{4}$
donc $C_f \cap (Ox) = \left\{\left(0;-\dfrac{1}{4}\right)\right\}$

pente de la tangente T à C_f au point d'abscisse 1:
$f'(1) = \dfrac{-4 \cdot 1 - 6}{(x-2)^3} = \dfrac{-10}{-1} = 10$

Exercice 2

a. $f(x) = x^2 \ln(1-x)$

condition: $1 - x > 0 \Leftrightarrow x < 1$ donc $D_f = D_{f'} =]-\infty; 1[$

$(\forall x \in D_{f'}) : f'(x) = 2x \cdot \ln(1-x) + x^2 \cdot \frac{-1}{1-x} = \frac{2x(1-x)\ln(1-x) - x^2}{1-x}$

b. $g(x) = 5(3e^x + 1)^4$

$D_f = D_{f'} = \mathbb{R}$

$(\forall x \in D_{f'}) : f'(x) = 5 \cdot 4(3e^x + 1)^3 \cdot 3e^x = 60e^x(3e^x + 1)^3$

Exercice 3

c: longueur du côté de la base en dm, h: hauteur de la boîte en dm.

quantité d'aluminium nécessaire: $A = 2c^2 + 4hc$ $\quad$ $(2 \cdot \text{base} + 4 \cdot \text{face latérale})$

volume de la boîte: $V = c^2 \cdot h = 8$ (car $1l = 1\ dm^3$) donc $h = \frac{8}{c^2}$.

finalement: $A(c) = 2c^2 + 4 \cdot \frac{8}{c^2} \cdot c = 2c^2 + \frac{32}{c}$ avec $c \in]0; +\infty[$

▸ $(\forall x \in]0; +\infty[) : A'(c) = 4c - \frac{32}{c^2} = \frac{4c^3 - 32}{c^2}$

▸ $A'(c) = 0 \Leftrightarrow 4c^3 - 32 = 0 \Leftrightarrow 4c^3 = 32 \Leftrightarrow c^3 = 8 \Leftrightarrow c = 2$

x	0		2		$+\infty$
A'	‖	$-$	0	$+$	
A	‖	↘	min	↗	

La quantité d'aluminium est minimale si $c = 2$ dm. Dans ce cas $h = \frac{8}{4} = 2$ dm.

Exercice 4

1. $(x-1)(2x^2 - 3x - 5) = 2x^3 - 3x^2 - 5x - 2x^2 + 3x + 5 = 2x^3 - 5x^2 - 2x + 5 \overset{!}{=} P(x)$

$$\begin{aligned} P(x) = 0 &\Leftrightarrow (x-1)(2x^2 - 3x - 5) \\ &\Leftrightarrow x - 1 = 0 \text{ ou } 2x^2 - 3x - 5 = 0 \\ &\Leftrightarrow x = 1 \qquad \Delta = 49, x_1 = \frac{3+7}{4} = \frac{5}{2}, x_2 = \frac{3-7}{4} = -1 \\ &S = \left\{1; \frac{5}{2}; -1\right\} \end{aligned}$$

2. points d'intersection: $f(x) = y_d$

$\Leftrightarrow 2x^3 - 5x^2 - x + 3 = x - 2$

$\Leftrightarrow 2x^3 - 5x^2 - 2x + 5 = 0$

$\Leftrightarrow x = 1$ ou $x = \frac{5}{2}$ ou $x = -1$ $\quad$ (d'après 1.)

Aire: $\int_{-1}^{1} (2x^3 - 5x^2 - 2x + 5)dx = \left[2 \cdot \frac{x^4}{4} - 5 \cdot \frac{x^3}{3} - 2 \cdot \frac{x^2}{2} + 5x\right]_{-1}^{1} = \left[\frac{x^4}{2} - \frac{5x^3}{3} - x^2 + 5x\right]_{-1}^{1}$

$= \left(\frac{1}{2} - \frac{5}{3} - 1 + 5\right) - \left(\frac{1}{2} + \frac{5}{3} - 1 - 5\right) = -\frac{10}{3} + 10 = \frac{20}{3}$

3. $5 + 2e^{3x} = 5e^{2x} + 2e^x$

$\Leftrightarrow 2e^{3x} - 5e^{2x} - 2e^x + 5 = 0$

Posons $y = e^x$.

on obtient: $2y^3 - 5y^2 - 2y + 5 = 0$

$\Leftrightarrow y = 1$ ou $y = \frac{5}{2}$ ou $y = -1$ $\quad$ (d'après 1.)

Revenons en arrière:

si $y = 1$, alors $e^x = 1 \Leftrightarrow x = 0$

si $y = \frac{5}{2}$, alors $e^x = \frac{5}{2} \Leftrightarrow x = \ln\frac{5}{2}$

si $y = -1$, alors $e^x = -1$, impossible!

$S = \left\{0; \ln\frac{5}{2}\right\}$

Exercice 5

$\ln(2x+5) = 2\ln x - \ln(2-x)$

conditions:

▸ $2x+5 > 0 \Leftrightarrow 2x > -5 \Leftrightarrow x > -\frac{5}{2}$

▸ $x > 0$

▸ $2-x > 0 \Leftrightarrow x < 2$

donc $D =]0;2[$

$\ln(2x+5) = 2\ln x - \ln(2-x)$
$\Leftrightarrow \ln(2x+5) + \ln(2-x) = 2\ln x$
$\Leftrightarrow \ln[(2x+5)(2-x)] = \ln x^2$
$\Leftrightarrow (2x+5)(2-x) = x^2$
$\Leftrightarrow 4x+10-2x^2-5x = x^2$
$\Leftrightarrow 3x^2+x-10 = 0$

$\Delta = 1+120 = 121; x_1 = \frac{-1-11}{6} = -2 \notin D; x_2 = \frac{-1+11}{6} = \frac{10}{6} = \frac{5}{3} \in D$

$S = \left\{\frac{5}{3}\right\}$

Exercice 6

1.a. $f(x) = \frac{2-x}{(x^2-4x+5)^2} = -\frac{1}{2}(2x-4)(x^2-4x+5)^{-2}$

$F(x) = -\frac{1}{2}\cdot\frac{(x^2-4x+5)^{-1}}{-1} = \frac{1}{2(x^2-4x+5)}$

b. $g(x) = 2e^{2x+3} - \frac{4}{3x-1} = 2e^{2x+3} - \frac{4}{3}\cdot\frac{3}{3x-1}$

$G(x) = e^{2x+3} - \frac{4}{3}\ln|3x-1|$

2. $I = \int_0^{\frac{\pi}{2}} 2\sin(3x)dx = \frac{2}{3}\int_0^{\frac{\pi}{2}} 3\sin(3x)dx = \left[\frac{2}{3}(-\cos(3x))\right]_0^{\frac{\pi}{2}} = 0 - \frac{2}{3}\cdot(-1) = \frac{2}{3}$

EXAMEN 2006 - T3BA/T3CC - Corrigé

Exercice 1

$f(x) = \frac{x^2-3}{2-x}$

a) condition: $2-x \neq 0 \Leftrightarrow x \neq 2$

donc $D_f = \mathbb{R}\setminus\{2\}$

b) ▸ $\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty}\frac{x^2}{-x} = \lim\limits_{x\to+\infty}(-x) = -\infty$

$\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty}\frac{x^2}{-x} = \lim\limits_{x\to-\infty}(-x) = +\infty$

▸ $\lim\limits_{x\to2} f(x) = \lim\limits_{x\to2}\frac{\overbrace{x^2-3}^{\to1}}{\underbrace{2-x}_{\to0}}$, il faut distinguer 2^+ et 2^-

$\lim\limits_{x\to2^+} f(x) = \lim\limits_{x\to2^+}\frac{\overbrace{x^2-3}^{\to1}}{\underbrace{2-x}_{\to0^-}} = -\infty$

$\lim\limits_{x\to2^-} f(x) = \lim\limits_{x\to2^-}\frac{\overbrace{x^2-3}^{\to1}}{\underbrace{2-x}_{\to0^+}} = +\infty$ A.V.: $x = 2$

c) $(\forall x\in\mathbb{R}\setminus\{2\}) : f'(x) = \frac{2x\cdot(2-x)-(x^2-3)(-1)}{(2-x)^2} = \frac{4x-2x^2+x^2-3}{(2-x)^2} = \frac{-x^2+4x-3}{(2-x)^2}$

d) $f'(x) = 0 \Leftrightarrow -x^2 + 4x - 3 = 0$

$\Delta = 16 - 12 = 4; x_1 = \dfrac{-4-2}{-2} = 3; x_2 = \dfrac{-4+2}{-2} = 1$

x	$-\infty$		1		2		3		$+\infty$
f'		$-$	0	$+$	$\Vert$	$+$	0	$-$	
f	$+\infty$	$\searrow$	-2	$\nearrow$	$+\infty \Vert -\infty$	$\nearrow$	-6	$\searrow$	$-\infty$

$f(1) = \dfrac{1-3}{2-1} = -2$ et $f(3) = \dfrac{9-3}{2-3} = \dfrac{6}{-1} = -6$

Exercice 2

a) $\ln(6x^2 + x - 2) - \ln(5 - 2x) = \ln(1 - x) - \ln 2$

conditions:

▸ $6x^2 + x - 2 > 0$

$\Delta = 1 + 24 = 25; x_1 = \dfrac{-1-5}{12} = -\dfrac{1}{2}; x_2 = \dfrac{-1+5}{12} = \dfrac{1}{3}$

x	$-\infty$		$-\frac{1}{2}$		$\frac{1}{3}$		$+\infty$
$6x^2 + x - 2$		$+$	0	$-$	0	$+$	

donc il faut que $x < -\dfrac{1}{2}$ ou $x > \dfrac{1}{3}$

▸ $5 - 2x > 0 \Leftrightarrow 2x < 5 \Leftrightarrow x < \dfrac{5}{2}$

▸ $1 - x > 0 \Leftrightarrow x < 1$

donc $D = \left]-\infty; -\frac{1}{2}\right[\cup \left]\frac{1}{3}; 1\right[$

$\ln(6x^2 + x - 2) - \ln(5 - 2x) = \ln(1 - x) - \ln 2$

$\Leftrightarrow \ln(6x^2 + x - 2) + \ln 2 = \ln(5 - 2x) + \ln(5 - 2x)$

$\Leftrightarrow \ln[2(6x^2 + x - 2)] = \ln[(5 - 2x)(1 - x)]$

$\Leftrightarrow 12x^2 + 2x - 4 = 5 - 5x - 2x + 2x^2$

$\Leftrightarrow 10x^2 + 9x - 9 = 0$

$\Delta = 81 + 360 = 441; x_1 = \dfrac{-9-21}{20} = -\dfrac{3}{2} \in D; x_2 = \dfrac{-9+21}{20} = \dfrac{3}{5} \in D$

$S = \left\{-\dfrac{3}{2}; \dfrac{3}{5}\right\}$

b) $(y+5)(3y^2 - 4y + 1) = 3y^3 - 4y^2 + y + 15y^2 - 20y + 5 = 3y^3 + 11y^2 - 19y + 5$

$3e^{3x} + 5 = 19e^x - 11e^{2x}$

$\Leftrightarrow 3e^{3x} + 11e^{2x} - 19e^x + 5 = 0$

Posons $y = e^x$

on obtient: $3y^3 + 11y^2 - 19y + 5 = 0$

$\Leftrightarrow (y+5)(3y^2 - 4y + 1) = 0$

$\Leftrightarrow y + 5 = 0$ ou $3y^2 - 4y + 1 = 0$

$\Leftrightarrow y = -5$ ou $\Delta = 16 - 12 = 4; x_1 = \dfrac{4+2}{6} = 1; x_2 = \dfrac{4-2}{6} = \dfrac{1}{3}$

Revenons en arrière:

si $y = -5$, alors $e^x = -5$ impossible!

si $y = 1$, alors $e^x = 1 \Leftrightarrow x = 0$

si $y = \frac{1}{3}$, alors $e^x = \frac{1}{3} \Leftrightarrow x = \ln\frac{1}{3}$

$S = \left\{0; \ln\frac{1}{3}\right\}$

Exercice 3

a) $f(x) = \ln\frac{2x+1}{x-2}$

condition: $\frac{2x+1}{x-2} > 0$

$2x+1 = 0 \Leftrightarrow x = -\frac{1}{2}$ et $x-2 = 0 \Leftrightarrow x = 2$

x	$-\infty$		$-\frac{1}{2}$		2		$+\infty$
$2x+1$		$-$	0	$+$		$+$	
$x-2$		$-$		$-$	0	$+$	
$\frac{2x+1}{x-2}$		$+$	0	$-$	$\parallel$	$+$	

donc $D_f = D_{f'} = \left]-\infty;-\frac{1}{2}\right[\cup \left]2;+\infty\right[$

$$(\forall x \in D_{f'}) : f'(x) = \frac{\frac{2(x-2)-(2x+1)}{(x-2)^2}}{\frac{2x+1}{x-2}} = \frac{2x-4-2x-1}{(x-2)^2} \cdot \frac{x-2}{2x+1} = \frac{-5}{(x-2)(2x+1)}$$

b) $f(x) = x - \ln(x^2+9)$

condition: $x^2+9 > 0$ toujours vrai!

donc $D_f = D_{f'} = \mathbb{R}$

$$(\forall x \in D_{f'}) : f'(x) = 1 - \frac{2x}{x^2+9} = \frac{x^2+9-2x}{x^2+9} = \frac{x^2-2x+9}{x^2+9}$$

c) $f(x) = e^{2x^2-x}(2x^3-5x)$

$D_f = D_{f'} = \mathbb{R}$

$(\forall x \in D_{f'}) : f'(x) = e^{2x^2-x}(4x-1)\cdot(2x^3-5x) + e^{2x^2-x}(6x^2-5)$

$= [(4x-1)(2x^3-5x)+(6x^2-5)]e^{2x^2-x} = (8x^4-20x^2-2x^3+5x+6x^2-5)e^{2x^2-x}$

$= (8x^4-2x^3-14x^2+5x-5)e^{2x^2-x}$

Exercice 4

a) $f(x) = 4x-5+\frac{5}{4x-5} = 4x-5+\frac{5}{4}\cdot\frac{4}{4x-5}$

$F(x) = 4\cdot\frac{x^2}{2} - 5x + \frac{5}{4}\ln|4x-5| + c = 2x^2 - 5x + \frac{5}{4}\ln|4x-5| + c, c \in \mathbb{R}$

b) $f(x) = \cos^5 x \sin x = -\cos^5 x \cdot (-\sin x)$

$F(x) = -\frac{\cos^6 x}{6} + c = -\frac{1}{6}\cos x + c, c \in \mathbb{R}$

Exercice 5

a)

$$I = \int_3^6 \frac{6}{\sqrt{3x+7}}dx = 2\int_3^6 3(3x+7)^{-\frac{1}{2}}dx = \left[2\frac{(3x+7)^{\frac{1}{2}}}{\frac{1}{2}}\right]_3^6 = \left[4\sqrt{3x+7}\,\right]_3^6 = 4\sqrt{25} - 4\sqrt{16} = 20-16 = 4$$

b) $$I = \int_{-1}^2 \frac{x-1}{(3x^2-6x+4)^2}dx = \frac{1}{6}\int_{-1}^2 (6x-6)(3x^2-6x+4)^{-2}dx = \left[\frac{1}{6}\cdot\frac{(3x^2-6x+4)^{-1}}{-1}\right]_{-1}^2$$

$$= \left[\frac{-1}{6(3x^2-6x+4)}\right]_{-1}^2 = \frac{-1}{6(12-12+4)} - \frac{-1}{6(3+6+4)} = -\frac{1}{24} + \frac{1}{78} = -\frac{78}{1872} + \frac{24}{1872}$$

$$= -\frac{54}{1872} = -\frac{3}{104}$$

Exercice 6

points d'intersection: $f(x) = g(x)$

$\Leftrightarrow -\frac{1}{2}x - 1 = -x^3 + 2x^2 + \frac{5}{2}x - 1$

$\Leftrightarrow x^3 - 2x^2 - 3x = 0$

$\Leftrightarrow x(x^2 - 2x - 3) = 0$

$\Leftrightarrow x = 0$ ou $x^2 - 2x - 3 = 0$

$\Delta = 4 + 12 = 16; x_1 = \frac{2+4}{2} = 3; x_2 = \frac{2-4}{2} = -1$

Aire: $\int_{-1}^{0}\left[\left(-\frac{1}{2}x - 1\right) - \left(-x^3 + 2x^2 + \frac{5}{2}x - 1\right)\right]dx + \int_{0}^{3}\left[\left(-x^3 + 2x^2 + \frac{5}{2}x - 1\right) - \left(-\frac{1}{2}x - 1\right)\right]dx$

$= \int_{-1}^{0}(x^3 - 2x^2 - 3x)dx + \int_{0}^{3}(-x^3 + 2x^2 + 3x)dx = \left[\frac{x^4}{4} - \frac{2x^3}{3} - \frac{3x^2}{2}\right]_{-1}^{0} + \left[-\frac{x^4}{4} + \frac{2x^3}{3} + \frac{3x^2}{2}\right]_{0}^{3}$

$= 0 - \left(\frac{1}{4} + \frac{2}{3} - \frac{3}{2}\right) + \left(-\frac{81}{4} + \frac{54}{3} + \frac{27}{2}\right) = \frac{-3 - 8 + 18 - 243 + 216 + 162}{12} = \frac{71}{6}$

EXAMEN 2006 - T3IF - Corrigé

Exercice 1

$f(x) = \frac{3x^2 - 4x}{(x-1)^2}$

a) condition: $(x-1)^2 \neq 0 \Leftrightarrow x - 1 \neq 0 \Leftrightarrow x \neq 1$

$D_f = \mathbb{R} - \{1\}$

b) $f(x) = \frac{3x^2 - 4x}{x^2 - 2x + 1}$

$$\blacktriangleright \left.\begin{array}{l} \lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty} \frac{3x^2}{x^2} = 3 \\ \text{de même : } \lim\limits_{x\to-\infty} f(x) = 3 \end{array}\right\} \text{A.H.: } y = 3$$

$$\blacktriangleright \lim_{x\to 1} \frac{\overbrace{3x^2 - 4x}^{\to -1}}{\underbrace{(x-1)^2}_{\to 0^+}} = -\infty \qquad \text{A.V.: } x = 1$$

c)

$(\forall x \in D_{f'}) : f'(x) = \frac{(6x-4)(x-1)^2 - (3x^2 - 4x)\cdot 2(x-1)}{(x-1)^4} = \frac{(x-1)[(6x-4)(x-1) - 2(3x^2 - 4x)]}{(x-1)^4}$

$= \frac{6x^2 - 4x - 6x + 4 - 6x^2 + 8x}{(x-1)^3} = \frac{4 - 2x}{(x-1)^3}$

d) $f'(x) = 0 \Leftrightarrow 4 - 2x = 0 \Leftrightarrow 2x = 4 \Leftrightarrow x = 2$

x	$-\infty$		1		2		$+\infty$
$4-2x$		$+$		$+$	0	$-$	
$(x-1)^3$		$-$	0	$+$		$+$	
$f'(x)$		$-$	$\|$	$+$	0	$-$	

$f(2) = \frac{12-8}{1} = 4$

x	$-\infty$		1		2		$+\infty$
f'		$-$	$\|$	$+$	0	$-$	
f	3	$\searrow$	$-\infty \;\|\; -\infty$	$\nearrow$	4	$\searrow$	3

e) ▸ $C_f \cap (Ox)$:

$f(x) = 0 \Leftrightarrow 3x^2 - 4x = 0 \Leftrightarrow x(3x-4) = 0$

$\Leftrightarrow x = 0$ ou $3x - 4 = 0$

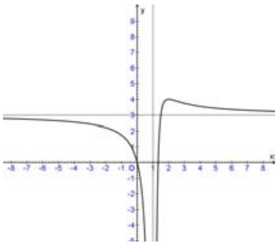
$\Leftrightarrow x = \dfrac{4}{3}$

donc $C_f \cap (Ox) = \{(0;0)\left(\dfrac{4}{3};0\right)\}$

▸ $C_f \cap (Oy)$:

$f(0) = 0$

donc $C_f \cap (Ox) = \{(0;0)\}$

f) 

x	-3	-1	$\frac{1}{2}$	3
f(x)	$\dfrac{39}{16}$	$\dfrac{7}{4}$	-5	$\dfrac{15}{4}$

Exercice 2

a) points d'intersection: $f(x) = g(x)$

$\Leftrightarrow x^2 + 2x - 2 = -x^2 - 2x + 4$

$\Leftrightarrow 2x^2 + 4x - 6 = 0$

$\Delta = 16 + 48 = 64; x_1 = \dfrac{-4+8}{4} = 1; x_2 = \dfrac{-4-8}{4} = -3$

b) aire: $\int_{-3}^{1}[(-x^2 - 2x + 4) - (x^2 + 2x - 2)]dx = \int_{-3}^{1}(-2x^2 - 4x + 6)dx = \left[-\dfrac{2x^3}{3} - \dfrac{4x^2}{2} + 6x\right]_{-3}^{1}$

$= \left(-\frac{2}{3} - 2 + 6\right) - \left(\frac{54}{3} - 18 - 18\right) = \dfrac{-2 + 12 + 54}{3} = \dfrac{64}{3}$

Exercice 3

a) $2\ln(x+1) - \ln(5-x) - \ln(2x+3) = 0$

conditions:

▸ $x + 1 > 0 \Leftrightarrow x > -1$

▸ $5 - x > 0 \Leftrightarrow x < 5$

▸ $2x + 3 > 0 \Leftrightarrow 2x > -3 \Leftrightarrow x > -\dfrac{3}{2}$

donc $D =]-1;5[$

$2\ln(x+1) - \ln(5-x) - \ln(2x+3) = 0$

$\Leftrightarrow 2\ln(x+1) = \ln(5-x) + \ln(2x+3)$

$\Leftrightarrow \ln\left[(x+1)^2\right] = \ln[(5-x)(2x+3)]$

$\Leftrightarrow (x+1)^2 = (5-x)(2x+3)$

$\Leftrightarrow x^2 + 2x + 1 = 10x + 15 - 2x^2 - 3x$

$\Leftrightarrow 3x^2 - 5x - 14 = 0$

$\Delta = 25 + 168 = 193; x_1 = \dfrac{5+\sqrt{193}}{6} \simeq 3,15 \in D; x_2 = \dfrac{5-\sqrt{193}}{6} \simeq -1,48 \notin D$

$S = \left\{\dfrac{5+\sqrt{193}}{6}\right\}$

b) $e^{2x} - 3e^x + 2 = 0$

posons: $y = e^x$

on obtient: $y^2 - 3y + 2 = 0$

$\Delta = 9 - 8 = 1; x_1 = \dfrac{3+1}{2} = 2; x_2 = \dfrac{3-1}{2} = 1$

Revenons en arrière:

si $y = 2$, alors $e^x = 2 \Leftrightarrow x = \ln 2$

si $y = 1$, alors $e^x = 1 \Leftrightarrow x = 0$

$S = \{0;\ln 2\}$

Exercice 4

$$f(x) = \frac{\ln(4-5x)}{5x-4}$$

conditions:

- $4 - 5x > 0 \Leftrightarrow 5x < 4 \Leftrightarrow x < \frac{4}{5}$
- $5x - 4 \neq 0 \Leftrightarrow x \neq \frac{4}{5}$

donc $D_f = D_{f'} =]-\infty; \frac{4}{5}[$

$$(\forall x \in D_{f'}) : f'(x) = \frac{\frac{-5}{4-5x} \cdot (5x-4) - \ln(4-5x) \cdot 5}{(5x-4)^2} = \frac{5 - 5\ln(4-5x)}{(5x-4)^2}$$

Exercice 5

a) $\int_0^1 \frac{8e^{4x}}{5+e^{4x}}dx = 2\int_0^1 \frac{4e^{4x}}{5+e^{4x}}dx = [2\ln|5+e^{4x}|]_0^1 = 2\ln(5+e^4) - 2\ln(5+1) = 2\ln(5+e^4) - 2\ln 6$

b)

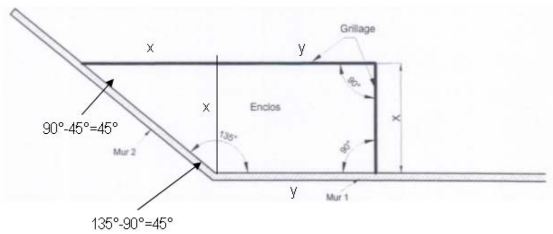
$$f(x) = \frac{(2\sqrt{x}-3)^2}{x \cdot \sqrt{x}} = \frac{4x - 12\sqrt{x} + 9}{x\sqrt{x}} = \frac{4x}{x\sqrt{x}} - \frac{12\sqrt{x}}{x\sqrt{x}} + \frac{9}{x\sqrt{x}} = \frac{4}{\sqrt{x}} - \frac{12}{x} + \frac{9}{x\sqrt{x}} = 4x^{-\frac{1}{2}} - \frac{12}{x} + 9x^{-\frac{3}{2}}$$

$$F(x) = 4\frac{x^{\frac{1}{2}}}{\frac{1}{2}} - 12\ln|x| + 9\frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} = 8x^{\frac{1}{2}} - 12\ln|x| - 18x^{-\frac{1}{2}} = 8\sqrt{x} - 12\ln|x| - \frac{18}{\sqrt{x}}$$

c) $g(x) = \frac{\sin(2x)}{\cos^2(2x)} = -\frac{1}{2}\cos^{-2}(2x)(-2\sin(2x))$

$$G(x) = -\frac{1}{2}\frac{\cos^{-1}(2x)}{-1} + c = -\frac{1}{2\cos(2x)} + c, c \in \mathbb{R}$$

Exercice 6



a)

surface du trapèze: $A = \frac{(x+y)+y}{2} \cdot x = \frac{(2y+x)x}{2}$ $\left(\text{Rappel: } \frac{\text{grande base} + \text{petite base}}{2} \cdot \text{hauteur}\right)$

b) longueur du grillage: $x + y + x = 9 \Leftrightarrow y = 9 - 2x$

Donc $A(x) = \frac{[2(9-2x)+x]x}{2} = \frac{(18-4x+x)x}{2} = \frac{18x-3x^2}{2} = -\frac{3}{2}x^2 + 9x$ avec $x \in \left]0; \frac{9}{2}\right[$

c) $\left(\forall x \in \left]0; \frac{9}{2}\right[\right) : A'(x) = -\frac{3}{2} \cdot 2x + 9 = -3x + 9$

$A'(x) = 0 \Leftrightarrow -3x + 9 = 0 \Leftrightarrow 3x = 9 \Leftrightarrow x = 3.$

x	0		3		$\frac{9}{2}$
A'	‖	$+$	0	0	
A	‖	↗	max	↘	

d) Finalement: $x = 3$ et $y = 9 - 2 \cdot 3 = 6$.

$A(3) = -\frac{3}{2} \cdot 9 + 27 = 27 - \frac{27}{2} = \frac{27}{2} = 13,5m^2$

EXAMEN 2006 - T3MG - Corrigé

Exercice 1

a) $f(x) = x^2 - \frac{1}{x^2} - \frac{x}{2} + \frac{2}{x} = x^2 - x^{-2} - \frac{x}{2} + \frac{2}{x}$

$F(x) = \frac{x^3}{3} - \frac{x^{-1}}{-1} - \frac{1}{2} \cdot \frac{x^2}{2} + 2\ln|x| + c = \frac{1}{3}x^3 + \frac{1}{x} - \frac{x^2}{4} + 2\ln|x| + c, c \in \mathbb{R}$

b) $f(x) = \left(\frac{x}{4} + 3\right)^4 = 4 \cdot \left(\frac{x}{4} + 3\right)^4 \cdot \frac{1}{4}$

$F(x) = 4 \cdot \frac{\left(\frac{x}{4} + 3\right)^5}{5} + c = \frac{4}{5}\left(\frac{x}{4} + 3\right)^5 + c, c \in \mathbb{R}$

c) $f(x) = \frac{\cos x}{\sin^3 x} = \sin^{-3} x \cdot \cos x$

$F(x) = \frac{\sin^{-2} x}{-2} + c = -\frac{1}{2\sin^2 x} + c, c \in \mathbb{R}$

d) $f(x) = \frac{(\ln x)^2}{x} = (\ln x)^2 \cdot \frac{1}{x}$

$F(x) = \frac{(\ln x)^3}{3} + c = \frac{1}{3}\ln^3 x + c, c \in \mathbb{R}$

Exercice 2

1) $\int_{-1}^{2}\left(2x + 1 + \frac{1}{2x+3}\right)dx = \int_{-1}^{2}\left(2x + 1 + \frac{1}{2} \cdot \frac{2}{2x+3}\right)dx = \left[\frac{2x^2}{2} + x + \frac{1}{2}\ln|2x+3|\right]_{-1}^{2}$

$= \left[x^2 + x + \frac{1}{2}\ln|2x+3|\right]_{-1}^{2} = \left(4 + 2 + \frac{1}{2}\ln 7\right) - \left(1 - 1 + \frac{1}{2}\ln 1\right) = 6 + \frac{1}{2}\ln 7$

2) $\int_{\ln 1}^{\ln 2} \frac{e^x}{1+e^x}dx = [\ln|1+e^x|]_{\ln 1}^{\ln 2} = \ln 3 - \ln 2$

3) $\int_{\frac{1}{2}}^{1} xe^{2x^2}dx = \frac{1}{4}\int_{\frac{1}{2}}^{1} 4xe^{2x^2}dx = \left[\frac{1}{4}e^{2x^2}\right]_{\frac{1}{2}}^{1} = \frac{1}{4}e^2 - \frac{1}{4}e^{\frac{1}{2}}$

Exercice 3

a) $f(x) = g(x)$

$\Leftrightarrow \frac{x^2}{2} + 1 = x + 5$

$\Leftrightarrow x^2 + 2 = 2x + 10$

$\Leftrightarrow x^2 - 2x - 8 = 0$

$\Delta = 4 + 32 = 36; x_1 = \frac{2+6}{2} = 4; x_2 = \frac{2-6}{2} = -2$

$g(4) = 9; g(-2) = 3$

donc $C_f \cap C_g = \{(-2;3);(4;9)\}$

b) aire:

$\int_{-2}^{3}\left[(x+5) - \left(\frac{x^2}{2} + 1\right)\right]dx = \int_{-2}^{3}\left(-\frac{x^2}{2} + x + 6\right)dx = \left[-\frac{1}{2} \cdot \frac{x^3}{3} + \frac{x^2}{2} + 6x\right]_{-2}^{3} = \left[-\frac{x^3}{6} + \frac{x^2}{2} + 6x\right]_{-2}^{3}$

$= \left(-\frac{27}{6} + \frac{9}{2} + 18\right) - \left(\frac{8}{6} + \frac{4}{2} - 12\right) = \frac{80}{3}$ unités d'aire

Exercice 4

$2\ln x = \ln\frac{1}{2} + \ln[2(6-x)]$

conditions:

▸ $x > 0$

▸ $2(6-x) > 0 \Longleftrightarrow 6-x > 0 \Longleftrightarrow x < 6$

donc $D =]0;6[$

$2\ln x = \ln\frac{1}{2} + \ln[2(6-x)]$

$\Longleftrightarrow \ln x^2 = \ln\left[\frac{1}{2}\cdot 2(6-x)\right]$

$\Longleftrightarrow x^2 = 6-x$

$\Longleftrightarrow x+x-6 = 0$

$\Delta = 1+24 = 25; x_1 = \dfrac{-1+5}{2} = 2 \in D; x_2 = \dfrac{-1-5}{2} = -3 \notin D$

$S = \{2\}$

Exercice 5

On considère le polynôme défini sur $\mathbb{R}$ par $P(x) = x^3 - 4x^2 + x + 6$.

1) $(x-2)(x^2-2x-3) = x^3 - 2x^2 - 3x - 2x^2 + 4x + 6 = x^3 - 4x^2 + x + 6 = P(x)$

2) $P(x) = 0$

$\Longleftrightarrow (x-2)(x^2-2x-3) = 0$

$\Longleftrightarrow x-2 = 0$ ou $x^2-2x-3 = 0$

$\Longleftrightarrow x = 2$ $\qquad \Delta = 4+12 = 16; x_1 = \frac{2+4}{2} = 3; x_2 = \frac{2-4}{2} = -1$

$S = \{2;3;-1\}$

3) $e^{3x} - 4e^{2x} + e^x + 6 = 0$.

Posons $y = e^x$

on obtient: $y^3 - y^2 + y + 6 = 0$

$\Longleftrightarrow y = 2$ ou $y = 3$ ou $y = -1$

Revenons en arrière:

si $y = 2$, alors $e^x = 2 \Longleftrightarrow x = \ln 2$

si $y = 3$, alors $e^x = 3 \Longleftrightarrow x = \ln 3$

si $y = -1$, alors $e^x = -1$ impossible!

$S = \{\ln 2; \ln 3\}$

Exercice 6

Volume $V = (\pi r^2)\cdot h = 3141 \Longleftrightarrow h = \dfrac{3141}{\pi r^2} = \dfrac{1000}{r^2}$ (avec $\pi = 3,141$)

Matériel utilisé: $A = \underbrace{(\pi r^2)}_{\text{fond}} + \underbrace{(2\pi r)\cdot h}_{\text{surface latérale}}$

on obtient finalement: $A(r) = \pi r^2 + 2\pi r\cdot\dfrac{1000}{r^2} = \pi r^2 + \dfrac{2000\pi}{r}$ $\qquad$ avec $r \in]0;+\infty[$

$(\forall x \in]0;+\infty[) : A'(x) = 2\pi r - \dfrac{2000\pi}{r^2} = \dfrac{2\pi r^3 - 2000\pi}{r^2}$

$A'(x) = 0 \Longleftrightarrow 2\pi r^3 - 2000\pi = 0 \Longleftrightarrow 2\pi r^3 = 2000\pi \Longleftrightarrow r^3 = \frac{2000\pi}{2\pi} = 1000 \Longleftrightarrow r = 10$

x	0		10		$+\infty$
A′	‖	−	0	+	
A	‖	↘	min	↗	

Donc le matériel à utiliser est minimal si $r = 10$ cm. Dans ce cas, $h = \dfrac{1000}{10^2} = 10$ cm