T3EE - Corrigé du devoir en classe de mathématiques I,2

Exercice 1

a) $f(x) = 2x^3 + \frac{5}{2}x^2 - 11x + \sqrt{3}$

$F(x) = 2 \cdot \frac{x^4}{4} + \frac{5}{2} \cdot \frac{x^3}{3} - 11 \cdot \frac{x^2}{2} + \sqrt{3}x + c = \boxed{\frac{1}{2}x^4 + \frac{5}{6}x^3 - \frac{11}{2}x^2 + \sqrt{3}x + c}, c \in \mathbb{R}$

b) $f(x) = \frac{-4}{(x-9)^3} = -4(x-9)^{-3}$

$F(x) = -4 \cdot \frac{(x-9)^{-2}}{-2} + c = 2(x-9)^{-2} + c = \boxed{\frac{2}{(x-9)^2} + c}, c \in \mathbb{R}$

c) $f(x) = (2x+3)(x^2+3x-5)^2$

$F(x) = \boxed{\frac{1}{3}(x^2+3x-5)^3 + c}, c \in \mathbb{R}$

d) $f(x) = (4x-3)(2x^2+1) = 8x^3 + 4x - 6x^2 - 3 = 8x^3 - 6x^2 + 4x - 3$

$F(x) = 8 \cdot \frac{x^4}{4} - 6 \cdot \frac{x^3}{3} + 4 \cdot \frac{x^2}{2} - 3x + c = \boxed{2x^4 - 2x^3 + 2x^2 - 3x + c}, c \in \mathbb{R}$

e) $f(x) = \frac{\sin x}{\cos^4 x} = -\sin x \cos^{-4} x \cdot (-1)$

$F(x) = -\frac{\cos^{-3} x}{-3} + c = \boxed{\frac{1}{3\cos^3 x} + c}, c \in \mathbb{R}$

f) $f(x) = (3x+4)^5 = \frac{1}{3} \cdot 3(3x+4)^5$

$F(x) = \frac{1}{3} \cdot \frac{(3x+4)^6}{6} + c = \boxed{\frac{1}{18}(3x+4)^6 + c}, c \in \mathbb{R}$

g) $f(x) = x \cdot \cos(2x^2+7) + \sin\frac{\pi}{6} = \frac{1}{4} \cdot 4x \cdot \cos(2x^2+7) + \frac{1}{2}$

$F(x) = \boxed{\frac{1}{4}\sin(2x^2+7) + \frac{1}{2}x + c}, c \in \mathbb{R}$

h) $f(x) = \frac{x-1}{\sqrt{x^2-2x+7}} = \frac{1}{2}(2x-2)(x^2-2x+7)^{-\frac{1}{2}}$

$F(x) = \frac{1}{2} \cdot \frac{(x^2-2x+7)^{\frac{1}{2}}}{\frac{1}{2}} + c = \boxed{\sqrt{x^2-2x+7} + c}, c \in \mathbb{R}$

Exercice 2

a) $f(x) = -6x(x^2-1)^2 = -3 \cdot 2x(x^2-1)^2$

$F(x) = -3 \cdot \frac{(x^2-1)^3}{3} + c = -(x^2-1)^3 + c, c \in \mathbb{R}$

$F(0) = 0 \Leftrightarrow -(-1)^3 + c = 0 \Leftrightarrow c = -1$

donc $\boxed{F(x) = -(x^2-1)^3 - 1}$

b) $f(x) = 6x^2 - \frac{4}{x^2} = 6x^2 - 4x^{-2}$

$F(x) = 6 \cdot \frac{x^3}{3} - 4 \cdot \frac{x^{-1}}{-1} + c = 2x^3 + \frac{4}{x} + c, c \in \mathbb{R}$

$F(-1) = -11 \Leftrightarrow -2 - 4 + c = -11 \Leftrightarrow c = -5$

donc $\boxed{F(x) = 2x^3 + \frac{4}{x} - 5}$

c) $f(x) = \cos\left(3x + \frac{\pi}{2}\right) = \frac{1}{3} \cdot 3\cos\left(3x + \frac{\pi}{2}\right)$

$F(x) = \frac{1}{3}\sin\left(3x + \frac{\pi}{2}\right) + c, c \in \mathbb{R}$

$F\left(\frac{\pi}{6}\right) = 1 \Leftrightarrow \frac{1}{3}\sin\left(\frac{\pi}{2} + \frac{\pi}{2}\right) + c = 1 \Leftrightarrow \frac{1}{3}\sin\pi + c = 1 \Leftrightarrow c = 1$

donc $\boxed{F(x) = \frac{1}{3}\sin\left(3x + \frac{\pi}{2}\right) + 1}$

Exercice 3

1° $f(x) = \frac{2x+4}{2x^2+3x-2}$

cond.: $2x^2 + 3x - 2 \neq 0 \left(\Delta = 9 + 16 = 25; x \neq \frac{-3-5}{4} = -2; x \neq \frac{-3+5}{4} = \frac{1}{2}\right)$

Donc $D_f = \mathbb{R} \setminus \left\{-2; \frac{1}{2}\right\}$

$\lim\limits_{x\to\pm\infty} f(x) = \lim\limits_{x\to\pm\infty} \frac{2x}{2x^2} = \lim\limits_{x\to\pm\infty} \frac{1}{x} = 0 \quad$ A.H.: $y = 0$

$\lim\limits_{x\to-2} \frac{\overbrace{2x+4}^{\to 0}}{\underbrace{2x^2+3x-2}_{\to 0}}$ il faut factoriser et simplifier par $(x+2)$

$2x + 4 = 2(x+2)$

$2x^2 + 3x - 2 = 2(x+2)\left(x - \frac{1}{2}\right) = (x+2)(2x-1)$

Donc $\lim\limits_{x\to-2} \frac{2x+4}{2x^2+3x-2} = \lim\limits_{x\to-2} \frac{2(x+2)}{(x+2)(2x-1)} = \lim\limits_{x\to-2} \frac{2}{2x-1} = \frac{2}{-5} = -\frac{2}{5}$

$\lim\limits_{x\to\frac{1}{2}} \frac{\overbrace{2x+4}^{\to 5}}{\underbrace{2x^2+3x-2}_{\to 0}}$ il faut calculer les limites à gauche et à droite

x	$-\infty$		-2		$\frac{1}{2}$		$+\infty$
$2x^2+3x-2$		$+$	0	$-$	0	$+$	

$\lim\limits_{x\to\frac{1}{2}^-} \frac{\overbrace{2x+4}^{\to 5}}{\underbrace{2x^2+3x-2}_{\to 0^-}} = -\infty$ et $\lim\limits_{x\to\frac{1}{2}^+} \frac{\overbrace{2x+4}^{\to 5}}{\underbrace{2x^2+3x-2}_{\to 0^+}} = +\infty \qquad$ A.V.: $x = \frac{1}{2}$

2° $f(x) = \frac{x^3}{(2x^2+7)^4}$

donc $D_f = D_{f'} = \mathbb{R}$

$(\forall x \in D_{f'}) : f'(x) = \frac{3x^2(2x^2+7)^4 - x^3 \cdot 4(2x^2+7)^3 \cdot 4x}{(2x^2+7)^8} = \frac{3x^2(2x^2+7)^4 - 16x^4(2x^2+7)^3}{(2x^2+7)^8}$

$= \frac{x^2(2x^2+7)^3[3(2x^2+7) - 16x^2]}{(2x^2+7)^8} = \frac{x^2(6x^2+21-16x^2)}{(2x^2+7)^5} = \frac{x^2(-10x^2+21)}{(2x^2+7)^5}$

3° $f(x) = x^3 - 3x^2 - 9x + 11$

$(\forall x \in D_{f'}) : f'(x) = 3x^2 - 6x - 9$

$f'(x) = 0 \Leftrightarrow 3x^2 - 6x - 9 = 0 \overset{:3}{\Leftrightarrow} x^2 - 2x - 3 = 0 \overset{\Delta=16}{\Leftrightarrow} x = \frac{2+4}{2} = 3$ ou $x = \frac{2-4}{2} = -1.$

x	$-\infty$		-1		3		$+\infty$
f'		$+$	0	$-$	0	$+$	
f	$-\infty$	$\nearrow$	16	$\searrow$	-16	$\nearrow$	$+\infty$

$f(-1) = -1 - 3 + 9 + 11 = 16$; $f(3) = 27 - 27 - 27 + 11 = -16$

$\lim\limits_{x\to\pm\infty} f(x) = \lim\limits_{x\to\pm\infty} x^3 = \pm\infty$