T3EE - Corrigé du devoir en classe de mathématiques II,1

Exercice 1

a) abscisses des points d'intersection:

$f(x) = g(x)$

$\Leftrightarrow -4x^2 + 5 = 1$

$\Leftrightarrow 4x^2 - 4 = 0$

$\Leftrightarrow x^2 - 1 = 0$

$\Leftrightarrow (x-1)(x+1) = 0$

$\Leftrightarrow x = 1$ ou $x = -1$

Donc $A = \int_{-1}^{1}[f(x) - g(x)]dx = \int_{-1}^{1}(-4x^2 + 4)dx$

$= \left[-4 \cdot \frac{x^3}{3} + 4x\right]_{-1}^{1} = \left(-\frac{4}{3} + 4\right) - \left(\frac{4}{3} - 4\right) = 8 - \frac{8}{3} = \frac{16}{3}$ u.a.

b) abscisses des points d'intersection:

$f(x) = g(x)$

$\Leftrightarrow x^2 = x^3 + x^2 - 4x$

$\Leftrightarrow x^3 - 4x = 0$

$\Leftrightarrow x(x^2 - 4) = 0$

$\Leftrightarrow x(x-2)(x+2) = 0$

$\Leftrightarrow x = 0$ ou $x = 2$ ou $x = -2$

Donc $A = \int_{-2}^{0}[(x^3 + x^2 - 4x) - x^2]dx + \int_{0}^{2}[x^2 - (x^3 + x^2 - 4x)]dx = \int_{-2}^{0}(x^3 - 4x)dx + \int_{0}^{2}(-x^3 + 4x)dx$

$= \left[\frac{x^4}{4} - 4 \cdot \frac{x^2}{2}\right]_{-2}^{0} + \left[-\frac{x^4}{4} + 4 \cdot \frac{x^2}{2}\right]_{0}^{2} = 0 - \left(\frac{16}{4} - \frac{16}{2}\right) + \left(-\frac{16}{4} + \frac{16}{2}\right) = -(-4) + 4 = 8$ u.a.

Exercice 2

a) $2\ln x = 3\ln 4$

condition: $x > 0$, donc $D =]0; +\infty[$

$2\ln x = 3\ln 4$

$\Leftrightarrow \ln(x^2) = \ln(4^3)$

$\Leftrightarrow x^2 = 64$

$\Leftrightarrow x = 8$ ou $x = -8 \notin D$

$S = \{8\}$

b) $\ln^2 x - 3\ln x = 10$

condition: $x > 0$, donc $D =]0; +\infty[$

$\ln^2 x - 3\ln x - 10 = 0$

posons $y = \ln x$

l'équation devient: $y^2 - 3y - 10 = 0$

$\Delta = 9 + 40 = 49; y_1 = \frac{3+7}{2} = 5; y_2 = \frac{3-7}{2} = -2$

Revenons en arrière:

si $y = 5$, alors $\ln x = 5 \Leftrightarrow x = e^5$

si $y = -2$, alors $\ln x = -2 \Leftrightarrow x = e^{-2}$

$S = \{e^5; e^{-2}\}$

c) $\ln(5x^2+3x) = \ln(3x+7) + \ln(4-x)$
conditions:
♦$5x^2+3x > 0$
$5x^2+3x = 0 \Leftrightarrow x(5x+3) = 0 \Leftrightarrow x = 0$ ou $x = -\frac{3}{5}$

x	$-\infty$		$-\frac{3}{5}$		0		$+\infty$
$5x^2+3x$		$+$	0	$-$	0	$+$	

donc il faut que $x < -\frac{3}{5}$ ou $x > 0$

♦$3x+7 > 0 \Leftrightarrow 3x > -7 \Leftrightarrow x > -\frac{7}{3}$

♦$4-x > 0 \Leftrightarrow x < 4$

finalement $D = \left]-\frac{7}{3};-\frac{3}{5}\right[\cup]0;4[$

$\ln(5x^2+3x) = \ln(3x+7) + \ln(4-x)$
$\Leftrightarrow \ln(5x^2+3x) = \ln[(3x+7)(4-x)]$
$\Leftrightarrow 5x^2+3x = -3x^2+5x+28$
$\Leftrightarrow -8x^2+2x+28 = 0$
$\Leftrightarrow 4x^2-x-14 = 0$
$\Delta = 225; x_1 = \dfrac{1+15}{8} = 2$; $x_2 = \dfrac{1-15}{8} = -\frac{7}{4}$
$S = \left\{-\frac{7}{4};2\right\}$

Exercice 3

1° $f(x) = \ln\left(\dfrac{2-x}{3+5x}\right)$
condition: $\dfrac{2-x}{3+5x} > 0$
$2-x = 0 \Leftrightarrow x = 2$ / $3+5x = 0 \Leftrightarrow x = -\frac{3}{5}$

x	$-\infty$		$-\frac{3}{5}$		2		$+\infty$
$2-x$		$+$		$+$	0	$-$	
$3+5x$		$-$	0	$+$		$+$	
$\frac{2-x}{3+5x}$		$-$	$\Vert$	$+$	0	$-$	

donc $D_f = D_{f'} =]-\frac{3}{5};2[$

$$(\forall x \in D_{f'}) : f'(x) = \frac{\frac{(-1)(3+5x)-(2-x)\cdot 5}{(3+5x)^2}}{\frac{2-x}{3+5x}} = \frac{\frac{-3-5x-10+5x}{(3+5x)^2}}{\frac{2-x}{3+5x}} = \frac{-13}{(3+5x)^2}\cdot\frac{3+5x}{2-x} = -\frac{13}{(3x+5)(2-x)}$$

2° $f(x) = 3\ln x + \ln^3 x + \ln(x^3) + \ln 3$
condition: $x > 0$, donc $D_f =]0;+\infty[= D_{f'}$

$$(\forall x \in D_{f'}) : f'(x) = 3\cdot\frac{1}{x} + 3\cdot\ln^2 x\cdot\frac{1}{x} + \frac{3x^2}{x^3} = \frac{3}{x} + \frac{3\ln^2 x}{x} + \frac{3}{x} = \frac{6+3\ln^2 x}{x}$$