T3EE - Corrigé du devoir en classe de mathématiques II,2

Exercice 1 (**3 + 4 + 4 + 4 + 5 + 4 = 24 points**)

a) $f(x) = 2x^4 + 3x^3 - \frac{1}{x}$

$F(x) = 2 \cdot \frac{x^5}{5} + 3 \cdot \frac{x^4}{4} - \ln|x| + c = \frac{2}{5}x^5 + \frac{3}{4}x^4 - \ln|x| + c, c \in \mathbb{R}$

b) $f(x) = \frac{3\sin x}{\cos^2 x} = 3\sin x \cos^{-2} x = -3\underbrace{(-\sin x)}_{u'}\underbrace{\cos^{-2} x}_{u^{-2}}$

$F(x) = -3\frac{\cos^{-1} x}{-1} + c = \frac{3}{\cos x} + c, c \in \mathbb{R}$

c) $f(x) = \frac{4x+1}{4x^2+2x-1} = \frac{1}{2} \cdot \frac{\overbrace{8x+2}^{u'}}{\underbrace{4x^2+2x-1}_{u}}$

$F(x) = \frac{1}{2}\ln|4x^2 + 2x - 1| + c, c \in \mathbb{R}$

d) $f(x) = \frac{4x^2+x+2}{2x} = \frac{4x^2}{2x} + \frac{x}{2x} + \frac{2}{2x} = 2x + \frac{1}{2} + \frac{1}{x}$

$F(x) = 2 \cdot \frac{x^2}{2} + \frac{1}{2}x + \ln|x| + c = x^2 + \frac{1}{2}x + \ln|x| + c, c \in \mathbb{R}$

e) $f(x) = x^2 \cdot e^{x^3} + x\sin(x^2) = \frac{1}{3} \cdot \underbrace{3x^2}_{u'} \cdot \underbrace{e^{x^3}}_{e^u} + \frac{1}{2} \cdot \underbrace{2x}_{u'}\underbrace{\sin(x^2)}_{\sin u}$

$F(x) = \frac{1}{3}e^{x^3} - \frac{1}{2}\cos(x^2) + c, c \in \mathbb{R}$

f) $f(x) = \frac{-4}{\sqrt{5x+7}} = -\frac{1}{5} \cdot 4 \cdot \underbrace{5}_{u'}\underbrace{(5x+7)^{-\frac{1}{2}}}_{u^{-\frac{1}{2}}}$

$F(x) = -\frac{4}{5}\frac{(5x+7)^{\frac{1}{2}}}{\frac{1}{2}} + c = -\frac{8}{5}\sqrt{5x+7} + c, c \in \mathbb{R}$

Exercice 2 (**6 + 6 + 4 = 16 points**)

a) $f(x) = \frac{\overbrace{e^x + 1}^{u}}{\underbrace{e^x - 2}_{v}}$

cond.:$e^x - 2 \neq 0 \Leftrightarrow e^x \neq 2 \Leftrightarrow x \neq \ln 2$

$D_f = D_{f'} = \mathbb{R} - \{\ln 2\}$

$(\forall x \in D_{f'}) : f'(x) = \frac{\overbrace{e^x}^{u'}\overbrace{(e^x-2)}^{v} - \overbrace{(e^x+1)}^{u}\overbrace{e^x}^{v'}}{\underbrace{(e^x-2)^2}_{v^2}} = \frac{e^{2x} - 2e^x - e^{2x} - e^x}{(e^x-2)^2} = \frac{-3e^x}{(e^x-2)^2}$

b) $f(x) = \frac{\ln x - x}{x^2}$

cond: $x > 0$ et $x \neq 0$

$D_f = D_{f'} =]0;+\infty[$

$(\forall x \in D_{f'}) : f'(x) = \frac{\overbrace{(\frac{1}{x} - 1)}^{u'} \cdot \overbrace{x^2}^{v} - \overbrace{(\ln x - x)}^{u} \cdot \overbrace{2x}^{v'}}{\underbrace{x^4}_{v^2}} = \frac{x - x^2 - 2x\ln x + 2x^2}{x^4} = \frac{x(x + 1 - 2\ln x)}{x^4} = \frac{x + 1 - 2\ln x}{x^3}$

c) $f(x) = \ln(e^x) + 2\sqrt{x} = x + 2\sqrt{x}$
cond:$x \geq 0$
$D_f = [0;+\infty[$ et $D_{f'} =]0;+\infty[$
$(\forall x \in D_{f'}) : f'(x) = 1 + 2 \cdot \dfrac{1}{2\sqrt{x}} = \dfrac{\sqrt{x}+1}{\sqrt{x}}$

Exercice 3 (**8** + (**3** + **5** + **4**) = **20 points**)
$\ln(2x+5) = 2\ln x - \ln(2-x)$

conditions:
♦ $2x+5 > 0 \Leftrightarrow 2x > -5 \Leftrightarrow x > -\frac{5}{2}$
♦ $x > 0$
♦ $2-x > 0 \Leftrightarrow x < 2$
donc $D =]0;2[$

$\ln(2x+5) = 2\ln x - \ln(2-x)$
$\Leftrightarrow \ln(2x+5) + \ln(2-x) = 2\ln x$
$\Leftrightarrow \ln[(2x+5)(2-x)] = \ln x^2$
$\Leftrightarrow (2x+5)(2-x) = x^2$
$\Leftrightarrow 4x + 10 - 2x^2 - 5x = x^2$
$\Leftrightarrow 3x^2 + x - 10 = 0$
$\Delta = 1 + 120 = 121; x_1 = \dfrac{-1-11}{6} = -2 \notin D; x_2 = \dfrac{-1+11}{6} = \dfrac{10}{6} = \dfrac{5}{3} \in D$
$S = \left\{\dfrac{5}{3}\right\}$

2° a) $P(x) = (x-1)(6x^2+x-1)$
$= 6x^3 + x^2 - x - 6x^2 - x + 1$
$\stackrel{!}{=} 6x^3 - 5x^2 - 2x + 1$

b) $P(x) = 0$
$\Leftrightarrow (x-1)(6x^2+x-1) = 0$
$\Leftrightarrow x - 1 = 0$ ou $6x^2 + x - 1 = 0$
$\Leftrightarrow x = 1 \qquad \Delta = 1 + 24 = 25; x_1 = \dfrac{-1+5}{12} = \dfrac{4}{12} = \dfrac{1}{3}; x_2 = \dfrac{-1-5}{12} = -\dfrac{6}{12} = -\dfrac{1}{2}$
$S = \left\{1;\dfrac{1}{3};-\dfrac{1}{2}\right\}$

c) $6e^{3x} - 5e^{2x} - 2e^x + 1 = 0$
Posons $y = e^x$
on obtient: $6y^3 - 5y^2 - 2y + 1 = 0$
d'après b): $y = 1$ ou $y = \dfrac{1}{3}$ ou $y = -\dfrac{1}{2}$
Revenons en arrière:
si $y = 1$ alors $e^x = 1 \Leftrightarrow x = \ln 1 = 0$
si $y = \dfrac{1}{3}$ alors $e^x = \dfrac{1}{3} \Leftrightarrow x = \ln\dfrac{1}{3} = -\ln 3$
si $y = -\dfrac{1}{2}$ alors $e^x = -\dfrac{1}{2}$ impossible
$S = \{0;-\ln 3\}$