T3EE - Corrigé I,2 du 11.12.2007

Exercice 1

a) $f(x) = x^8 - 7x^4 + \frac{3}{2}x^2 - \pi$

$$F(x) = \frac{x^9}{9} - 7 \cdot \frac{x^5}{5} + \frac{3}{2} \cdot \frac{x^3}{3} - \pi x + c$$
$$= \frac{1}{9}x^9 - \frac{7}{5}x^5 + \frac{1}{2}x^3 - \pi x + c \qquad ,c \in \mathbb{R}$$

b) $f(x) = (3x+1)(3x^2+2x)^3 = \frac{1}{2}\underbrace{(6x+2)}_{u'}\underbrace{(3x^2+2x)^3}_{u^3}$

$$F(x) = \frac{1}{2} \cdot \frac{(3x^2+2x)^4}{4} + c$$
$$= \frac{1}{8}(3x^2+2x)^4 + c \qquad ,c \in \mathbb{R}$$

c) $f(x) = \frac{3}{(2x+7)^3} = 3 \cdot \frac{1}{2} \cdot \underbrace{2}_{u'}\underbrace{(2x+7)^{-3}}_{u^{-3}}$

$$F(x) = \frac{3}{2} \cdot \frac{(2x+7)^{-2}}{-2} + c$$
$$= \frac{-3}{4(2x+7)^2} + c \qquad ,c \in \mathbb{R}$$

d) $f(x) = \sin x \cdot \cos^5 x = -\underbrace{(-\sin x)}_{u'} \cdot \underbrace{\cos^5 x}_{u^5}$

$$F(x) = -\frac{\cos^6 x}{6} + c$$
$$= -\frac{1}{6}\cos^6 x + c \quad ,c \in \mathbb{R}$$

e) $f(x) = (5x+1)(5x^2+2) = 25x^3 + 5x^2 + 10x + 2$

$$F(x) = 25 \cdot \frac{x^4}{4} + 5 \cdot \frac{x^3}{3} + 10 \cdot \frac{x^2}{2} + 2x + c$$
$$= \frac{25}{4}x^4 + \frac{5}{3}x^3 + 5x^2 + 2x + c \qquad ,c \in \mathbb{R}$$

f) $f(x) = \frac{-x}{\sqrt{x^2 - \frac{1}{3}}} = -\frac{1}{2} \cdot \underbrace{2x}_{u'}\underbrace{\left(x^2 - \frac{1}{3}\right)^{-\frac{1}{2}}}_{u^{-\frac{1}{2}}}$

$$F(x) = -\frac{1}{2} \cdot \frac{\left(x^2 - \frac{1}{3}\right)^{\frac{1}{2}}}{\frac{1}{2}} + c$$
$$= -\sqrt{x^2 - \frac{1}{3}} + c \qquad ,c \in \mathbb{R}$$

g) $f(x) = \underbrace{2x}_{u'} \cdot \underbrace{\cos(x^2)}_{\cos u} + \frac{\pi}{6}\tan\left(\frac{\pi}{6}\right)$

$$F(x) = \sin(x^2) + \frac{\pi}{6}\tan\left(\frac{\pi}{6}\right)x + c \quad ,c \in \mathbb{R}$$

Exercice 2

1° $f(x) = 3x^2 + 4x - 3$ avec $F(-2) = -3$

$$\begin{aligned} F(x) &= 3 \cdot \tfrac{x^3}{3} + 4 \cdot \tfrac{x^2}{2} - 3x + c \\ &= x^3 + 2x^2 - 3x + c \qquad ,c \in \mathbb{R} \end{aligned}$$

$F(-2) = -3$
$\Leftrightarrow (-2)^3 + 2(-2)^2 - 3(-2) + c = -3$
$\Leftrightarrow -8 + 8 + 6 + c = -3$
$\Leftrightarrow c = -3 - 6 = -9$

donc $F(x) = x^3 + 2x^2 - 3x - 9$

2°a) $\int_1^2 \left(\frac{x^3}{2} - \frac{2}{x^3}\right)dx$
$= \int_1^2 \left(\frac{1}{2}x^3 - 2x^{-3}\right)dx$
$= \left[\frac{1}{2} \cdot \frac{x^4}{4} - 2 \cdot \frac{x^{-2}}{-2}\right]_1^2$
$= \left[\frac{1}{8}x^4 + \frac{1}{x^2}\right]_1^2$
$= \left(\frac{16}{8} + \frac{1}{4}\right) - \left(\frac{1}{8} + \frac{1}{1}\right)$
$= \frac{16}{8} + \frac{2}{8} - \frac{1}{8} - \frac{8}{8}$
$= \frac{9}{8}$

b) $\int_1^6 \frac{2}{\sqrt{4x+1}}dx$
$= \int_1^6 \frac{1}{2} \cdot 4(4x+1)^{-\frac{1}{2}}dx$
$= \left[\frac{1}{2} \cdot \frac{(4x+1)^{\frac{1}{2}}}{\frac{1}{2}}\right]_1^6$
$= \left[\sqrt{4x+1}\right]_1^6$
$= \sqrt{25} - \sqrt{5}$
$= 5 - \sqrt{5}$

Exercice 3

a) $f(x) = \dfrac{1-2x}{(3x+1)^2}$

cond.: $(3x+1) \neq 0 \Leftrightarrow x \neq -\dfrac{1}{3}$ donc $D_f = D_{f'} = \mathbb{R} - \left\{-\frac{1}{3}\right\}$

$\left(\forall x \in \mathbb{R} - \left\{-\frac{1}{3}\right\}\right) : f'(x) = \dfrac{-2(3x+1)^2 - (1-2x) \cdot 2(3x+1) \cdot 3}{(3x+1)^4}$

$= \dfrac{(3x+1)[-2(3x+1) - 6(1-2x)]}{(3x+1)^4} = \dfrac{-6x - 2 - 6 + 12x}{(3x+1)^3} = \dfrac{6x-8}{(3x+1)^3}$

b) $f(x) = 2x\cos(x^2) + \dfrac{\pi}{6}\tan\left(\dfrac{\pi}{6}\right)$

$D_f = D_{f'} = \mathbb{R}$

$(\forall x \in \mathbb{R}) : f'(x) = 2\cos(x^2) + 2x \cdot (-\sin(x^2)) \cdot 2x = 2\cos(x^2) - 4x^2\sin(x^2)$

c) $f(x) = (4x+1)\sqrt{2x+1}$

cond.: $2x + 1 \geq 0 \Leftrightarrow x \geq -\frac{1}{2}$ donc $D_f = \left[-\frac{1}{2};+\infty\right[$ et $D_{f'} = \left]-\frac{1}{2};+\infty\right[$

$(\forall x \in D_{f'}) : f'(x) = 4\sqrt{2x+1} + (4x+1)\dfrac{2}{2\sqrt{2x+1}}$

$= 4\sqrt{2x+1} + \dfrac{4x+1}{\sqrt{2x+1}} = \dfrac{4(2x+1) + 4x + 1}{\sqrt{2x+1}} = \dfrac{12x+5}{\sqrt{2x+1}}$